1. If \(1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + 30 \cdot 32 = A\), then \(1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 + \cdots + 30 \cdot 33\) is
   (a) \(A + 180\)  (b) \(A + 240\)  (c) \(A + 320\)  (d) \(A + 336\)  (e) \(A + 465\)

2. If it takes 5 sheep 5 days to eat all of the grass on 5 football fields, how many days will it take 3 sheep to eat all of the grass on 3 football fields?
   (a) 2  (b) 3  (c) 4  (d) 5  (e) 6

3. Which of the following is a simplification of \(\frac{\cos 5x + \cos 3x}{\sin 5x - \sin 3x}\)?
   (a) \(\tan x\)  (b) \(\cot x\)  (c) \(\tan 2x\)  (d) \(\cot 2x\)  (e) \(\tan 2x / \tan x\)

4. Compute the volume of the set of points, \((x, y, z)\), in three dimensional space satisfying \(0 < x < 1, 0 < y < 1, 0 < z < 1\), and \(x < y < z\).
   (a) \(1/3\)  (b) \(1/4\)  (c) \(1/5\)  (d) \(1/6\)  (e) \(1/8\)

5. The equation \(x^{x^{\sqrt{x}}} = (x^{\sqrt{x}})^x\) has two positive solutions. One obvious solution is \(x = 1\). Which of the following corresponds to the other solution?
   (a) \(1/2\)  (b) \(2/3\)  (c) \(9/4\)  (d) \(3/2\)  (e) None of these
6. Which of the following is a simplification of \( \left( \frac{1 \cdot 2 \cdot 4 + 2 \cdot 4 \cdot 8 + \cdots + n \cdot 2n \cdot 4n}{1 \cdot 3 \cdot 9 + 2 \cdot 6 \cdot 18 + \cdots + n \cdot 3n \cdot 9n} \right)^{\frac{1}{2}} \)?

(a) 1/3 (b) 2/3 (c) 4/9 (d) 8/27 (e) None of these

7. Evaluate \( \tan 10^\circ \cdot \tan 20^\circ \cdot \tan 30^\circ \cdot \tan 40^\circ \cdot \tan 50^\circ \cdot \tan 60^\circ \cdot \tan 70^\circ \cdot \tan 80^\circ \).

(a) 0 (b) 1/2 (c) 1/\sqrt{3} (d) \sqrt{3} (e) 1

8. If \( f(x) + 2f(1-x) = x^2 \) for all \( x \), then \( 3f(x) = \).

(a) \( x^2 \) (b) \( x^2 - 4x + 2 \) (c) \( x^2 - 2x + 1 \) (d) \( x^2 + 2x - 1 \) (e) None of these

9. Solve the inequality \( \sqrt{\log_2 x - 1} + \frac{1}{2} \log_{\frac{1}{3}} x^3 + 2 > 0 \).

(a) \( x \in [2, 3] \) (b) \( x \in (2, 3] \) (c) \( x \in [2, 4) \) (d) \( x \in (2, 4] \) (e) None of these

10. Find the value of \( \sqrt{5 + 2\sqrt{6}} - \sqrt{5 - 2\sqrt{6}} \).

(a) \( 4\sqrt{6} \) (b) \( 2\sqrt{2} \) (c) \( 2\sqrt{3} \) (d) \( \frac{3\sqrt{3}}{2} \) (e) None of these
11. If \( \log_x y + \log_y x = 10 \), then what is the value of \((\log_x y)^2 + (\log_y x)^2\)?

(a) 94  (b) 96  (c) 98  (d) 100  (e) None of these

12. If the distance between the centers of two unit circles is \( \sqrt{3} \), what is the area of their intersection?

(a) \( \frac{\pi}{3} - \frac{1}{2} \)  (b) \( \frac{\pi}{3} + \frac{1}{2} \)  (c) \( \frac{\pi}{3} - \frac{\sqrt{3}}{2} \)  (d) \( \frac{\pi}{3} + \frac{\sqrt{3}}{2} \)  (e) none of these

13. Suppose that \( 0 < x < \frac{\pi}{2} \), \( a = \ln(\sin x) \), and \( b = \ln(\cos x) \). Express \( a \) in terms of \( b \).

(a) \( \frac{\ln(1+e^{ib})}{2} \)  (b) \( \frac{\ln(1-e^{ib})}{2} \)  (c) \( \frac{\ln(e^{ib})-1}{2} \)  (d) \( \frac{\ln(1+e^{2b})}{2} \)  (e) \( \ln(1-e^{2b}) \)

14. Twelve identical circles are all tangent to the same unit circle. If each of these twelve circles is tangent to exactly two of the other eleven circles (see picture), what is the radius of each of the twelve circles?

\( \frac{1+\sin 15^\circ}{1-\sin 15^\circ} \)  (b) \( \frac{\sin 15^\circ}{1+\sin 15^\circ} \)  (c) \( \frac{\cos 15^\circ}{1-\cos 15^\circ} \)  (d) \( \frac{1+\cos 15^\circ}{1-\cos 15^\circ} \)  (e) \( \sin 15^\circ \)

15. Find the sum of \( 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + 2012 \cdot 2012! \)

(a) 2013!-1  (b) 2013!  (c) 2013!+1  (d) 2013!+2  (e) None of these
16. If three positive integers $a_1, a_2$ and $a_3$ satisfy the equation $\log_2 6 = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{x}}}$, then $(a_1, a_2, a_3)$ is

(a) (2, 1, 1)  (b) (2, 2, 1)  (c) (2, 1, 2)  (d) (2, 2, 2)  (e) none of these

17. If $x_1$ and $x_2$ are the two solutions to the equation $x^2 - x + 1 = 0$, what is the value of $x_1^{2012} + x_2^{2012}$?

(a) 0  (b) 1  (c) -1  (d) $\sqrt{3}$  (e) None of these

18. If $0 < b < 1$, $0 < a < \pi/4$, $x = (\sin a)^{\log_b \sin a}$, $y = (\cos a)^{\log_b \cos a}$, and $z = (\sin a)^{\log_b \cos a}$, then the relationship between $x$, $y$ and $z$ is

(a) $z < x < y$  (b) $x < y < z$  (c) $x < z < y$  (d) $y < z < x$  (e) None of these

19. The integer $2^{23} + 2^{22} - 2^{21} - 2^{20} + 2^x$ is a perfect square for some nonnegative integer $x$. How many zeros are at the end of this perfect square?

(a) 0  (b) 2  (c) 4  (d) 6  (e) 8

20. What is the coefficient of $x_1^3x_2^2x_4$ in the expansion of $(x_1 + x_2 + x_3 + x_4)^6$?

(a) 48  (b) 54  (c) 60  (d) 64  (e) None of these
21. Let $a, b,$ and $c$ be positive real numbers. Determine the largest possible total number of real roots for the three following polynomials: $ax^2 + bx + c$, $bx^2 + cx + a$, and $cx^2 + ax + b$.

(a) 3  (b) 4  (c) 5  (d) 6  (e) None of these

22. A class of 10 students took a math test. Each problem was solved by exactly 7 of the students. If the first nine students each solved 4 problems, how many problems did the tenth student solve?

(a) 2  (b) 3  (c) 4  (d) 5  (e) 6

23. What is the remainder if $(999^{777})^{333}$ is divided by 11?

(a) 9  (b) 5  (c) 2  (d) 1  (e) 0

24. Let $z = i^{11} + i^{22} + \cdots + i^{2012} + i^{2012}$. Find $|z|$.

(a) 1  (b) $\sqrt{2}$  (c) $\sqrt{3}$  (d) 2  (e) None of these

25. Suppose $n(n + 1)a_{n+1} = n(n - 1)a_n - (n - 2)a_{n-1}$ for all positive integers $n$, $a_0 = 1$, and $a_1 = 2$. Find the value of

$$\frac{a_0}{a_1} + \frac{a_1}{a_2} + \cdots + \frac{a_{50}}{a_{51}}$$

(a) 1327  (b) $2655/2$  (c) 1328  (d) $2657/2$  (e) None of these