Moments and Centers of Mass

Moments and Center of Mass: One-Dimensional System

Let the point masses $m_1, m_2, \ldots, m_n$ be located at $x_1, x_2, \ldots, x_n$.

1. The moment about the origin is $M_0 = m_1x_1 + m_2x_2 + \cdots + m_nx_n$.

2. The center of mass is $\overline{x} = \dfrac{M_0}{m}$, where $m = m_1 + m_2 + \cdots + m_n$ is the total mass of the system.

Moments and Center of Mass: Two-Dimensional System

Let the point masses $m_1, m_2, \ldots, m_n$ be located at $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$.

1. The moment about the $y-$axis is $M_y = m_1x_1 + m_2x_2 + \cdots + m_nx_n$.

2. The moment about the $x-$axis is $M_x = m_1y_1 + m_2y_2 + \cdots + m_ny_n$.

3. The center of mass $(\overline{x}, \overline{y})$ is

   $$\overline{x} = \frac{M_y}{m} \text{ and } \overline{y} = \frac{M_x}{m}$$

   where $m = m_1 + m_2 + \cdots + m_n$ is the total mass of the system.

Moments and Center of Mass of a Planar Lamina

Let f and g be continuous functions such that $f(x) \geq g(x)$ on $[a, b]$ and consider the planar lamina of uniform density ρ bounded by the graphs of $y = f(x), y = g(x)$, and $a \cdot \ \ x \cdot \ \ b$.

1. The moments about the $x-$ and $y-$axes are

   $$M_x = \frac{\rho}{2} \int_a^b [f(x) + g(x)][f(x) - g(x)] \, dx$$

   $$M_y = \rho \int_a^b x[f(x) - g(x)] \, dx$$

2. The center of mass $(\overline{x}, \overline{y})$ is given by

   $$\overline{x} = \frac{M_y}{m} \text{ and } \overline{y} = \frac{M_x}{m}$$

   where $m = \rho \int_a^b [f(x) - g(x)] \, dx$ is the total mass of the lamina.