

Solving Trigonometric Equations

Find all solutions a.) in $[0, 2\pi)$ and b.) in general for the following trigonometric equations.

Problem Set 1

- 1.) $\cos \theta = -\frac{1}{2}$
- 2.) $\tan \theta = 1$
- 3.) $2 \sin x + \sqrt{3} = 0$
- 4.) $2 \sin x + 1 = 0$
- 5.) $2 \cos x = \sqrt{2}$
- 6.) $\cos x + 1 = 0$
- 7.) $\sec \beta = 2$
- 8.) $\csc \alpha = -1$
- 9.) $\tan \theta = \sqrt{3}$
- 10.) $\tan^2 \beta = 1$

Problem Set 2

- 1.) $\sin 4x = \frac{\sqrt{3}}{2}$
- 2.) $\cos 3x = -\frac{1}{2}$
- 3.) $\sec 3\beta = 2$
- 4.) $2 \cos 2\theta - \sqrt{3} = 0$
- 5.) $\sqrt{3} \tan\left(\frac{t}{3}\right) = 1$
- 6.) $2 \sin 3\theta + \sqrt{2} = 0$
- 7.) $\cos\left(\frac{x}{4}\right) = -\frac{\sqrt{2}}{2}$
- 8.) $\sqrt{3} + 2 \sin 2\beta = 0$

Problem Set 3

- 1.) $(\cos \alpha - 1)(\sin \alpha + 1) = 0$
- 2.) $(2 \sin \beta + 1)(2 \cos \beta + 3) = 0$
- 3.) $(2 \sin u - 1)(\cos u - \sqrt{2}) = 0$
- 4.) $\tan \alpha + \tan^2 \alpha = 0$
- 5.) $2 \sin^2 x = 1 - \sin x$
- 6.) $2 \cos^2 t + 3 \cos t + 1 = 0$
- 7.) $\tan^2 x \sin x = \sin x$
- 8.) $4 \sin^2 x \tan x - \tan x = 0$

Problem Set 1

Problem	Solution(s) in $[0, 2\pi)$	General Solution
1.	$\theta = \frac{2\pi}{3}$ $\theta = \frac{4\pi}{3}$	$\theta = \frac{2\pi}{3} + 2k\pi$ $\theta = \frac{4\pi}{3} + 2k\pi$
2.	$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$	$\theta = \frac{\pi}{4} + k\pi$
3.	$x = \frac{4\pi}{3}$ $x = \frac{5\pi}{3}$	$x = \frac{4\pi}{3} + 2k\pi$ $x = \frac{5\pi}{3} + 2k\pi$
4.	$x = \frac{7\pi}{6}$ $x = \frac{11\pi}{6}$	$x = \frac{7\pi}{6} + 2k\pi$ $x = \frac{11\pi}{6} + 2k\pi$
5.	$x = \frac{\pi}{4}$ $x = \frac{7\pi}{4}$	$x = \frac{\pi}{4} + 2k\pi$ $x = \frac{7\pi}{4} + 2k\pi$
6.	$x = \pi$	$x = \pi + 2k\pi$
7.	$\beta = \frac{\pi}{3}$ $\beta = \frac{5\pi}{3}$	$\beta = \frac{\pi}{3} + 2k\pi$ $\beta = \frac{5\pi}{3} + 2k\pi$
8.	$\alpha = \frac{3\pi}{2}$	$\alpha = \frac{3\pi}{2} + 2k\pi$
9.	$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$	$\theta = \frac{\pi}{3} + k\pi$
10.	$\beta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	$\beta = \frac{\pi}{4} + k\frac{\pi}{2}$

Problem Set 2

Problem	Solution(s) in $[0, 2\pi)$	General Solution
1.	$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$ $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$	$x = \frac{\pi}{12} + k\frac{\pi}{2}$ $x = \frac{\pi}{6} + k\frac{\pi}{2}$
2.	$x = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}$ $x = \frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}$	$x = \frac{2\pi}{9} + \frac{2k\pi}{3}$ $x = \frac{4\pi}{9} + \frac{2k\pi}{3}$
3.	$\beta = \frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}$ $\beta = \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$	$\beta = \frac{\pi}{9} + \frac{2k\pi}{3}$ $\beta = \frac{5\pi}{9} + \frac{2k\pi}{3}$
4.	$\theta = \frac{\pi}{12}, \frac{13\pi}{12}$ $\theta = \frac{11\pi}{12}, \frac{23\pi}{12}$	$\theta = \frac{\pi}{12} + k\pi$ $\theta = \frac{11\pi}{12} + k\pi$
5.	$t = \frac{\pi}{2}$	$t = \frac{\pi}{2} + 2k\pi$
6.	$\theta = \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{21\pi}{12}$ $\theta = \frac{7\pi}{12}, \frac{15\pi}{12}, \frac{23\pi}{12}$	$\theta = \frac{5\pi}{12} + \frac{2k\pi}{3}$ $\theta = \frac{7\pi}{12} + \frac{2k\pi}{3}$
7.	No solutions in $[0, 2\pi)$	$x = 3\pi + 8k\pi$ $x = 5\pi + 8k\pi$
8.	$\beta = \frac{2\pi}{3}, \frac{5\pi}{3}$ $\beta = \frac{5\pi}{6}, \frac{11\pi}{6}$	$\beta = \frac{2\pi}{3} + k\pi$ $\beta = \frac{5\pi}{6} + k\pi$

Problem Set 3

Problem	Solution(s) in $[0, 2\pi)$	General Solution
1.	$\alpha = 0$ $\alpha = \frac{3\pi}{2}$	$\alpha = 2k\pi$ $\alpha = \frac{3\pi}{2} + 2k\pi$
2.	$\beta = \frac{7\pi}{6}$ $\beta = \frac{11\pi}{6}$	$\beta = \frac{7\pi}{6} + 2k\pi$ $\beta = \frac{11\pi}{6} + 2k\pi$
3.	$u = \frac{\pi}{6}$ $u = \frac{5\pi}{6}$	$u = \frac{\pi}{6} + 2k\pi$ $u = \frac{5\pi}{6} + 2k\pi$
4.	$\alpha = 0, \pi$ $\alpha = \frac{3\pi}{4}, \frac{7\pi}{4}$	$\alpha = k\pi$ $\alpha = \frac{3\pi}{4} + k\pi$
5.	$x = \frac{\pi}{6}$ $x = \frac{5\pi}{6}$ $x = \frac{3\pi}{2}$	$x = \frac{\pi}{6} + 2k\pi$ $x = \frac{5\pi}{6} + 2k\pi$ $x = \frac{3\pi}{2} + 2k\pi$

Problem Set 3

Problem	Solution(s) in $[0, 2\pi)$	General Solution
6.	$t = \frac{2\pi}{3}$ $t = \frac{4\pi}{3}$ $t = \pi$	$t = \frac{2\pi}{3} + 2k\pi$ $t = \frac{4\pi}{3} + 2k\pi$ $t = \pi + 2k\pi$
7.	$x = 0, \pi$ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	$x = k\pi$ $x = \frac{\pi}{4} + \frac{k\pi}{2}$
8.	$x = 0, \pi$ $x = \frac{\pi}{6}, \frac{7\pi}{6}$ $x = \frac{5\pi}{6}, \frac{11\pi}{6}$	$x = k\pi$ $x = \frac{\pi}{6} + k\pi$ $x = \frac{5\pi}{6} + k\pi$