

Math Contest Level 2 - March 6, 2015

Coastal Carolina University

- Which of one of the following points is on an asymptote to the hyperbola $16x^2 - 9y^2 = 144$?
a) (16, 9) b) (12, 16) c) (9, 4) d) (9, 16) e) none of these
- The area of a rectangle is 9 square inches. P and Q are adjacent vertices of the rectangle. If R is the point at which the diagonals of the rectangle meet, then the degree measure of $\angle PRQ$ is 60° . The length of PQ is
a) $3^{1/4}$ b) $3^{1/3}$ c) $3^{1/2}$ d) $3^{3/4}$ e) none of these
- What is the value of x if $2^{2^x} = 16^{16^{16}}$?
a) 63 b) 64 c) 65 d) 66 e) none of these
- What is the coefficient of x^4 in the expansion of $(x - 2)^7$?
a) 35 b) -35 c) 280 d) -280 e) none of these
- If p people can do a job in d days, then $p + q$ people can do the job in how many days?
a) $d + q$ b) $d - q$ c) $\frac{pd}{p + q}$ d) $\frac{d}{p + q}$ e) none of these

6. What is the exact value of $\log_6(7) \cdot \log_5(6) \cdot \log_7(8) \cdot \log_4(5)$?

- a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\ln 2$ d) 2 e) none of these

7. When the polynomial $5x^4 + 4x^3 + 3x^2 + Mx + N$ is divided by $x^2 + 1$, the remainder is 0. The value of $N - M$ is

- a) -6 b) -2 c) 2 d) 6 e) none of these

8. How many zeros are at the end of the number $2015!$ if it is written in base 15?

- a) 250 b) 251 c) 500 d) 501 e) none of these

9. Which of these is equal to $\frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$?

- a) $\frac{\sqrt[3]{x^2} + \sqrt[3]{x} + 1}{\sqrt{x} + 1}$ b) $\frac{\sqrt{x} + 1}{\sqrt[3]{x^2} + \sqrt[3]{x} + 1}$ c) $\frac{\sqrt[3]{x} + 1}{\sqrt{x} + 1}$ d) $\frac{\sqrt{x} + 1}{\sqrt[3]{x} + 1}$ e) none of these

10. Two fair six sided dice are rolled repeatedly. What is the probability that a sum of 5 will occur before a sum of 8?

- a) $\frac{4}{9}$ b) $\frac{5}{11}$ c) $\frac{6}{13}$ d) $\frac{7}{15}$ e) none of these

11. The fraction $\frac{\tan^2 \theta + \cos^2 \theta}{\sec \theta - \sin \theta}$ simplifies to:
a) $\sec \theta - \sin \theta$ b) $\sec \theta + \sin \theta$ c) $\tan \theta - \cos \theta$ d) $\tan \theta + \cos \theta$ e) none of these
12. The solution of the equation $\log_2(\log_3(\log_4(3 \cdot 5^{4x-10} - 11))) = 0$ is
a) $x = 2$ b) $x = 4$ c) $x = 6$ d) $x = 8$ e) none of these
13. The sequence 3, 9, 12, 27, 30, 36, 39, ... is the increasing sequence of the sums of the distinct powers of 3. What term is the number $6561 = 3^8$?
a) 128^{th} b) 144^{th} c) 169^{th} d) 243^{rd} e) none of these
14. Mr. Jones drives from home to work at an average speed of 45 mph. His average speed from work back home is 50 mph. What is his average speed for the round-trip from home to work and back?
a) 47.5 mph b) $15\sqrt{10}$ mph c) $\frac{900}{19}$ mph d) 48 mph e) none of these
15. Suppose A and B are points on the circle with center C . The angle $\angle ACB$ is 30° . If a point D is randomly chosen on the circle, then what is the probability that the triangle $\triangle ADB$ is obtuse?
a) $1/12$ b) $1/2$ c) $3/4$ d) $5/6$ e) none of these

16. The product of the the solutions of the equation $2(\log_2 x)^2 - \log_2 x^{17} + 8 = 0$ is
- a) -3 b) $128\sqrt{2}$ c) $256\sqrt{2}$ d) 4 e) none of these
17. Suppose that r is a root of the polynomial $x^4 - x^3 + x^2 - x + 1$. What is $r^{20} - r^{15} + r^{10} - r^5 + 1$?
- a) -1 b) 1 c) -5 d) 5 e) none of these
18. The expression $\sqrt{3}\cos(x) + \sin(x)$ can be rewritten as
- a) $\sin(x + \frac{\pi}{6})$ b) $2\sin(\frac{2\pi}{3} - x)$ c) $\cos(\frac{7\pi}{6} - x)$ d) $2\cos(x + \frac{\pi}{3})$ e) none of these
19. Let $f(x) = \frac{x}{1+x}$ and let $g(x) = \frac{rx}{1-x}$. Let S be the set of all real numbers r such that $f(g(x)) = g(f(x))$ for infinitely many real numbers x . The number of elements in the set S is
- a) 0 b) 1 c) 2 d) ∞ e) none of these
20. Circle C_1 with center P_1 and circle C_2 with center P_2 intersect in points Q and R such that $\angle QP_1R = 30^\circ$ and $\angle QP_2R = 60^\circ$. What is the ratio of the area of circle C_1 to the area of circle C_2 ?
- a) $\frac{1}{2 - \sqrt{3}}$ b) $\frac{3}{1 - \sqrt{2}}$ c) $3 + \sqrt{2}$ d) $7 + 4\sqrt{3}$ e) none of these

21. Evaluate $\cos(20^\circ) \cdot \cos(40^\circ) \cdot \cos(80^\circ)$.
- a) $1/2$ b) $1/4$ c) $1/8$ d) $1/16$ e) none of these
22. What is the next term of the sequence 1, 11, 21, 1211, 111221, ...?
- a) 112221 b) 122211 c) 313112 d) 312211 e) none of these
23. How many ways can 6 people be seated at a round table if two of them refuse to sit next to each other?
- a) 72 b) 120 c) 144 d) 720 e) none of these
24. Suppose that the digits of an integer N consist of the integers from 1 to 100 written in order from left to right, so that $N = 123456789101112 \dots 9899100$. what is the remainder when N is divided by 11?
- a) 2 b) 5 c) 8 d) 10 e) none of these
25. Circle A with radius 1 is externally tangent to circle B which has radius 4 at the point P . The circle A rolls around the circumference of circle B without slipping and returns to the point P . How many revolutions does circle A make?
- a) 3 b) 4 c) 5 d) 6 e) none of these

26. Let $p(x)$ be the cubic polynomial $5x^3 - 6x^2 + K$. If the three roots of $p(x)$ form an arithmetic progression, then the value of K is

- a) $3/4$ b) $4/5$ c) $16/25$ d) $25/36$ e) none of these

27. Evaluate $\cos(20^\circ) \cdot \cos(40^\circ) \cdot \cos(80^\circ)$.

- a) $1/2$ b) $1/4$ c) $1/8$ d) $1/16$ e) none of these

28. What is the largest prime factor of 2014?
a) 3 b) 13 c) 17 d) 19 E) none of these
29. Joel collects comic books as a hobby. At a comic book convention he bought a Superman comic book and then sold it immediately for 20% more than the amount he paid for it. Next, he bought a Spiderman comic book and sold it immediately for 20% less than its purchase price. he received the same amount for the Superman and the Spiderman comic books. Which one of the following amounts could represent the total Joel paid for the two comic books?
a) \$30.30 b) \$30.45 c) \$30.60 d) \$30.75 e) \$30.90 none of these
30. It takes Agatha 90 minutes to mow her mother's yard and her brother Barnabus can mow it in 60 minutes. How long would it take them to mow it together using two lawn mowers.
A) 36 minutes b) 150 minutes c) 40 minutes d) 42 minutes e) None of these
31. There are 3 red balls, 2 green balls, and 3 purple balls in a bag. If you take out 3 balls at once, what is the probability that you took out at least one red ball?
a) 1 b) $\frac{27}{28}$ c) $\frac{25}{28}$ D) $\frac{23}{28}$ e) none of these
32. Define a sequence $a_1 = 1$, $a_{n+1} = 6a_n + 1$. What is a_{2014} ?
a) 1 b) 3 c) 5 d) 7 e) none of these
33. What is the value of $\log_2(\log_2(\log_2 16))$?
a) 0 B) 1 c) 2 d) 3 e) none of these
34. If $\sin \theta \cos \theta = \frac{1}{4}$ and $-90^\circ \leq \theta \leq 90^\circ$, then $\theta =$
a) 10° B) 15° c) 25° d) 30° e) none of these