Math Contest Level 2 - March 17, 2017

Coastal Carolina University

1. Which one of the following numbers is NOT prime?

   a) 241 b) 247 c) 251 d) 257 e) 263

2. If the sum goes on forever, what does $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$ equal?

   a) 1 b) 2 c) 3 d) 4 e) other

3. A primitive Pythagorean triple is a set of natural numbers (a, b, c) such that $a^2+b^2 = c^2$ and the three numbers are pairwise relatively prime. If we restrict to the cases where $c < 100$, which of the following is NOT a possible value for a or b?

   a) 12 b) 13 c) 14 d) 15 e) 16

4. We define a positive integer q to be *almost prime* if it has exactly one positive divisor other that 1 and itself. Compute the sum of the first 4 numbers which are almost prime.

   a) 83 b) 85 c) 87 d) 89 e) other

5. If a finite, connected, planar graph G is drawn without any edge intersections and there are 12 vertices and 16 edges, how many faces are there?

   a) 2 b) 4 c) 5 d) 6 e) other

6. We can construct the "negaFibonacci" numbers by using the recurrence relation: $F_{n-2} = F_n - F_{n-1}$ with seeds $F_{-2} = 1$ and $F_{-1} = 0$. Which of the following is NOT a negaFibonacci number?

   a) 89 b) 144 c) 233 d) -377 e) other

7. A magician presents two doors to you. The sign on Door 1 says, "Behind this door, there is an adorable kitten, and behind the other is a ferocious tiger." The sign on Door 2 says, "Behind one of these doors, there is an adorable kitten, and behind the other there is a ferocious tiger." The magician then says that one of these signs is true and the other is false. Which of the following must be true?

   a) Door 1 has a tiger, and Door 2 has a kitten. b) Door 1 has a kitten, and Door 2 has a tiger. c) Both doors have kittens. d) Both doors have tigers. e) other

8. How many zeros are at the end of the number 2017! if it is converted to base 16?

   a) 126 b) 133 c) 500 d) 502 e) other

9. What is the coefficient of the x^5y^3 term if you expand $(x + y)^8$?

   a) 36 b) 56 c) 70 d) 84 e) other

10. Three angles A, B and C, placed in standard position have terminal sides passing through the points (1,1), (1, 2) and (1,3) respectively. Find the sum of the measures of the three angles.

a) 160 b) 165 c) 170 d) 175 e) other

11. What is the last digit of the number $7^{7^{7^7}}$?

a) 1 b) 3 c) 5 d) 7 e) 9

12. A recursive sequence is given as follows: $a_1 = a_2 = 1$ and $a_{n+1} = \frac{1+a_n}{a_{n-1}}$. What is a_{2017}?

a) 0 b) 1 c) 2 d) 3 e) other

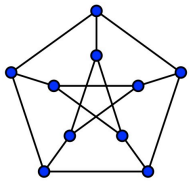
13. If 20 people show up for the Math Club meeting and everyone shakes hands with everyone else, how many handshakes happened in total?

a) 110 b) 190 c) 210 d) 380 e) other

14. Albert and Bernard now want to know how old Cheryl is. Cheryl: I have two younger brothers. The product of all our ages is 144. Albert: We still don't know your age. What other hints can you give us? Cheryl: The sum of all our ages is the bus number of this bus that we are on. Bernard: Of course we know the bus number, but we still don't know your age. Cheryl: Oh, I forgot to tell you that my brothers are twins. Albert and Bernard: Oh, now we know your age. So what is Cheryl's age?

a) 4 b) 9 c) 16 d) 36 e) other

15. The chromatic number of a graph G is the smallest number of colors needed to color the vertices of G so that no two adjacent vertices share the same color. What is the chromatic number of the following graph?



a) 1 b) 2 c) 3 d) 4 e) other

16. Two congruent circles centered at points A and B each pass through the other circle's center. The line containing both A and B is extended to intersect the circles at points C and D. The circles intersect at two points, E and F. What is the measure of $\angle CED$?

a) 105° b) 120° c) 135° d) 150° e) other

17. Find the value of n such that $\log_2(3) \cdot \log_3(4) \cdot \log_4(5) \cdot \ldots \cdot \log_n(n+1) = 10$.

a) 255 b) 511 c) 1023 d) 2047 e) other

18. A bug in the corner of the ceiling of a room spots food on the floor in the diagonally opposite corner of the room. If the room is 10 ft x 12 ft with an 8 ft ceiling, what is the shortest distance the bug can crawl to reach the food?

a) $10\sqrt{5}$ b) $4\sqrt{29}$ c) $8 + 2\sqrt{61}$ d) $2\sqrt{117}$ e) other

19. Find $f(0) + f(1) + f(2) + f(3)$ if f is an odd function with period 4.

a) -1 b) 0 c) 1 d) 4 e) other

20. At a track meet, 213 sprinters enter a 100-meter dash competition. A maximum of six sprinters can compete at one time. At the end of each race, the non-winners are eliminated, and the winner will compete again in a later race. What is the minimum number of races that are needed to determine the champion sprinter?

a) 36 b) 42 c) 43 d) 60 e) other

21. A drawer contains 1 white, 2 black, 3 green and 4 red socks. Socks are drawn at random, one at a time until two of the same color are selected. What is the probability that 5 selections are required to obtain the matching pair?

a) 4/35 b) 8/105 c) 1/210 d) 36/625 e) other

22. Joel collects comic books as a hobby. At a comic book convention he bought a Superman comic book and then sold it immediately for 20% more than the amount he paid for it. Next, he bought a Spiderman comic book and sold it immediately for 20% less than its purchase price. He received the same amount for the Superman and the Spiderman comic books. Which one of the following amounts could represent the total Joel paid for the two comic books?

a) \$30.30 b) \$30.45 c) \$30.60 d) \$30.75 e) other

23. What is the correct ordering of the numbers $2^{51}, 3^{33}$, and 4^{24}?

a) $2^{51} > 3^{33} > 4^{24}$ b) $2^{51} > 4^{24} > 3^{33}$ c) $3^{33} > 4^{24} > 2^{51}$ d) $3^{33} > 2^{51} > 4^{24}$ e) other

24. Evaluate $\sin(10°) \cdot \sin(30°) \cdot \sin(50°) \cdot \sin(70°) \cdot \sin(90°)$.

a) $1/8$ b) $\sqrt{3}/8$ c) $1/16$ d) $\sqrt{3}/16$ e) other

25. What is the inverse of the function $f(x) = \frac{e^x + e^{-x}}{2}, x \geq 0$?

a) $\ln(x + \sqrt{x^2 - 1})$ b) $\ln(x - \sqrt{x^2 - 1})$ c) $\ln(4x - 3)$
d) $\ln(-\frac{12}{5}x^2 + \frac{47}{5}x - 6)$ e) other