Math Contest Level 2 - March 2, 2018

Coastal Carolina University

1. What is the 2018th digit of the number 12345678910111213...?

   a) 0 b) 7 c) 8 d) 9 e) other

2. Find the value of c so that the equation $|x^2 + 8x + 12| = c$ has exactly 3 solutions.

   a) 2 b) 3 c) 4 d) 5 e) other

3. A circle of radius r is inscribed in an equilateral triangle. Find the area of the triangle.

   a) $2\sqrt{3}r^2$ b) $4\sqrt{2}r^2$ c) $3\sqrt{3}r^2$ d) $4\sqrt{3}r^2$ e) other

4. What is the largest postage that cannot be made using 6, 9 and 20-cent stamps?

   a) 8 cents b) 19 cents c) 28 cents d) 37 cents e) other

5. $\cos(15^\circ) - \sin(15^\circ) =$

   a) $\frac{1}{2}$ b) $\frac{\sqrt{2}}{2}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{\sqrt{1+\sqrt{3}}}{4}$ e) other

6. Circles of radius 10 and 17 intersect at 2 points. The chord made by these two points has length 16. List all possible values for the distance between their centers.

a) 9 and 21 b) 10 and 20 c) 11 and 19 d) 12 and 18 e) other

7. $\log(225!) - \log(223!) = 1 + \log(n!)$. Find n.

a) 6 b) 7 c) 8 d) 9 e) other

8. Let $f(x, y)$ be defined by, $f(x, 0) = x$ and $f(x, y + 1) = f(f(x, y), y)$. Which of the following is the largest?

a) $f(10, 15)$ b) $f(11, 14)$ c) $f(12, 13)$ d) $f(13, 12)$ e) $f(14, 11)$

9. How many zeros are at the end of the number 2018! if it is written in base 72?

a) 670 b) 671 c) 505 d) 504 e) other

10. Each day, Cheryl takes the train from work arriving at the station at 6:00pm. Her partner Taylor works from home and leaves everyday at 5:00 to pick Cheryl up at station. One day, Cheryl gets to the train station at 5:00 and begins to walk home. Taylor meets her en-route and they return home together at 6:40. How long did Cheryl walk?

a) 40 min b) 45 min c) 50 min d) 55 min e) other

11. Twenty-five people are sitting in a circle. Three are chosen at random. What is the probability that no two of them were sitting next to each other?

a) 23/36 b) 25/39 c) 29/42 d) 35/46 e) other

12. Let $n = \underline{a}\ \underline{b}\ \underline{c}$ be a three digit number with positive digits a, b and c. Suppose

$$f(n) = a + b + c + ab + ac + bc + abc.$$

How many such three digit integers n satisfy $f(n) = n$?

a) 6 b) 7 c) 8 d) 9 e) other

13. John has 2018 coins. They are all pure silver except for one of them which is an alloy and weighs a little bit more than the pure coins. John wishes to determine which of the coins is the alloy coin using a balance scale. What is the minimum number of weighings needed to guarantee that he will be able to determine the alloy coin?

a) 6 b) 7 c) 10 d) 11 e) other

14. $6^{83} + 8^{83} \equiv k (\text{mod } 49)$. Find k.

a) 0 b) 20 c) 28 d) 48 e) other

15. If $f(m+1) = m(-1)^{m+1} - 2f(m)$ and $f(1) = f(2019)$, find the value of

$$f(1) + f(2) + f(3) + \cdots + f(2018).$$

a) -1009/3 b) -1009/2 c) -505/3 d) -505/2 e) other

16. Find n if $\arctan(1/3) + \arctan(1/4) + \arctan(1/5) + \arctan(1/n) = \pi/4$.

a) 40 b) 41 c) 42 d) 43 e) other

17. The number of real number solutions to the equation $(x^2 + 2x)^{x^2-3x+2} = 1$.

a) 2 b) 3 c) 4 d) 5 e) other

18. Let x and y be randomly chosen numbers between 0 and 1. What is the probability that $\dfrac{y-x}{y+x}$ is closest to an odd integer?

a) 1/5 b) 1/4 c) 1/3 d) 1/2 e) other

19. A circle has parallel chords of lengths 12 and 16 that are 7 units apart. A third chord is drawn halfway between the other two. What is the length of this third chord?

a) $\sqrt{165}$ b) $\sqrt{193}$ c) $\sqrt{221}$ d) $\sqrt{249}$ e) other

20. Let two standard, well shuffled, 52 card decks be placed one on top of the other. If we pick up each card one at a time from the top deck and counted the number of cards between that card and it's matching card in the second deck, i.e. the ace of spades to the other ace of spades, what would the sum of the numbers be?

a) 2652 b) 5226 c) 2256 d) 2526 e) other

21. The number of integers between 1000 and 9999 which are perfect squares whose digits sum to 14.

a) 0 b) 1 c) 9 d) 14 e) other

22. Let $p(x)$ be a polynomial of degree 5 with integer coefficients and at least one integer root. If $p(2) = 13$ and $p(10) = 5$ then which of these must be a root of the polynomial?

a) 0 b) 6 c) 12 d) 15 e) other

23. Find $x + y$ if (x, y) is the positive solution pair to the system:

$$x^2 + x\sqrt[3]{xy^2} = 208$$

$$y^2 + y\sqrt[3]{yx^2} = 1053$$

a) 32 b) 35 c) 36 d) 40 e) other

24. Circles of radius 1, 3 and r lie externally tangent to each other and internally tangent to a circle of radius 4. Find r.

a) 11/12 b) 12/13 c) 13/14 d) 14/15 e) other

25. If the angles A, B and C of a triangle are successive terms in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then what is the value of the expression:

$$\frac{a}{c}\sin(2C) + \frac{c}{a}\sin(2A)?$$

a) 0 b) $\sqrt{2}$ c) $\sqrt{3}$ d) 1 e) other