

39th ANNUAL DR. SUBHASH C. SAXENA MATH CONTEST

March 2, 2018

COASTAL CAROLINA UNIVERSITY

Notes and directions.

- * Do not turn this page over until you are told to do so.
- * Fill in the SCANTRON form according to your proctor's instructions.
- * Calculators are not permitted on this test.
- * You have fifty minutes to complete the test. If you finish early, you should leave quietly and proceed to the Hicks Dining Hall for lunch.
- * The test is yours to keep, so use any extra space for scratch work.

Good luck!

Math Contest - Level 1

March 2, 2018

1) Solve for x : $\sqrt{49 + \sqrt{x}} = 8$

- A. 64 B. 49 C. 100 D. 225 E. 15

2) Which of the points given below is on the graph of the equation

$$x^3 - y^2 + 2xy + 12x = 5$$

- A. $(-1, -2)$ B. $(1, -2)$ C. $(2, 1)$ D. $(-1, 2)$ E. $(-2, -1)$

3) A math book has 504 regularly numbered pages. On how many pages does the digit 6 appear in the page number?

- A. 95 B. 97 C. 78 D. 79 E. 80

4) Any three members of the Chants painting team, working together, can paint eight rooms in four days. If you need 20 rooms to be painted in fifteen days, how many Chants should you hire?

- A. 1 B. 2 C. 3 D. 5 E. 8

5) How many diagonals does a convex dodecagon have (A dodecagon has 12 sides. The sides don't count as diagonals)?

- A. 66 B. 132 C. 10 D. 51 E. 54

- 6) The measure of each interior angle of a regular polygon is nine times that of an exterior angle of the polygon. How many sides does the polygon have?
- A. 20 B. 22 C. 25 D. 18 E. 16
- 7) Find the vertex of the parabola $y = 3x^2 - 4x + 5$
- A. $(\frac{4}{3}, 5)$ B. $(-\frac{2}{3}, \frac{89}{9})$ C. $(-\frac{4}{3}, \frac{143}{9})$ D. $(4, 37)$ E. $(\frac{2}{3}, \frac{33}{9})$
- 8) Simplify $\frac{2^{2018} - 2^{2017}}{128}$
- A. 2^{2010} B. 64^{2017} C. 2^{1001} D. $\frac{1}{64}$ E. $2^{2016} - 2^{2015}$
- 9) Express the repeating decimal $0.4\overline{56} = 0.456565656\dots$ as a fraction in lowest terms
- A. $\frac{42}{90}$ B. $\frac{52}{90}$ C. $\frac{255}{990}$ D. $\frac{452}{990}$ E. $\frac{42}{99}$
- 10) Find the remainder when $(x + 7)^3$ is divided by $(x + 2)$
- A. 729 B. -729 C. 0 D. 125 E. 25
- 11) 20% of students at a university who major in math also major in education. If 1% of the students at that university major in math and 10% major in education, what percentage of students at that university major in both math and education?
- A. 2% B. 0.02% C. 0.2% D. 0.1% E. 1%
- 12) \overline{PQ} is a diameter of a circle in which \overline{PR} is a chord; S is a point on \overline{PQ} such that $RS \perp \overline{PQ}$. If $\overline{PS} = 25$ cm and $\overline{QS} = 4$ cm, how long is \overline{RS} ?
- A. 9 cm B. 10 cm C. 21 cm D. 3 cm E. 7 cm

- 13) The number of square feet in the total surface area of a right circular cylinder is equal to the number of cubic feet in its volume. If the radius of its base is four times its altitude, what is its volume?
- A. $250\pi ft^3$ B. $520\pi ft^3$ C. $25\pi ft^3$ D. $52\pi ft^3$ E. $100\pi ft^3$
- 14) In a recursive sequence, if $a_1 = 3$, $a_2 = 5$ and $a_{n+2} = a_{n+1} - a_n$ then what is the value of a_{2018} ?
- A. 2 B. 5 C. 3 D. -3 E. -5
- 15) Which of the following polynomials has only integer roots?
- A. $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$ B. $2x^5 + 5x^4 + 5x - 1$ C. $3x^5 + 10x^3 + 10x^2 + 5x - 1$
D. $2x^5 + 5x^4 + 10x^3 - 5x + 1$ E. $-2x^5 - 5x^4 + 10x^3 + 10x^2 + 5x - 1$
- 16) $\overline{AB} = 4\sqrt{3}$ and $\overline{CD} = 4$ are two chords on a circle with center O . It is known that the ratio of the distance from O to \overline{AB} and the distance from O to \overline{CD} is $1 : \sqrt{3}$. What is the radius of the circle?
- A. 1 B. $\sqrt{2}$ C. 4 D. 3 E. $\sqrt{5}$.
- 17) If $1 + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \dots = 2018$ and $\frac{1}{c} + \frac{1}{c^2} + \frac{1}{c^3} + \dots = 2016$ where $|a| > 1$ and $|c| > 1$, then what is the ratio $\frac{c}{a}$?
- A. $\frac{2015 \times 2017}{2016 \times 2018}$ B. $\frac{2015 \times 2018}{2016 \times 2017}$ C. $\frac{2016 \times 2018}{2015 \times 2017}$ D. $\frac{2015 \times 2017}{2014 \times 2016}$ E. $\frac{2017^2}{2017^2 - 1}$
- 18) In how many ways can one choose 6 cells from a 4×4 table such that three cells lie in the same row or column?
- A. 216 B. 432 C. 324 D. 162 E. 234
- 19) A person rolls five fair six-sided dice, what is the probability that the person rolls exactly one 1 and exactly one 2?
- A. $\frac{40}{243}$ B. $\frac{60}{243}$ C. $\frac{40}{81}$ D. $\frac{60}{81}$ E. $\frac{10}{81}$.

- 20) If x is a real number such that $x - x^{-1} = 5$, what is the value of $x^3 - x^{-3}$?
- A. 27 B. $10 + 7\sqrt{15}$ C. 125 D. $7 + 10\sqrt{15}$ E. 140.
- 21) Let S be a sphere of radius $9m$ and let C be a cube whose eight vertices lie on the surface of S . What is the volume of C in m^3 ?
- A. $81\sqrt{3}$ B. $81\sqrt{6}$ C. $162\sqrt{3}$ D. $648\sqrt{3}$ E. $162\sqrt{2}$
- 22) In how many ways can six people be seated in a circle if two arrangements are considered the same whenever each person has the same neighbors(not necessarily on the same side)?
- A. 40 B. 60 C. 80 D. 100 E. 120
- 23) Let PQR be a right triangle with a right angle at Q . Draw the segment \overline{QS} so that it bisects \overline{PR} . If $m\angle QPS = 25^\circ$, what is $m\angle QSR$?
- A. 50° B. 65° C. 155° D. 56° E. 130°
- 24) A person travels from Point X to Point Y with an average speed of 21 mph and returns on the same path with an average speed of 42 mph. What is the average speed for this entire round trip in mph?
- A. 31.5 B. 30 C. 32 D. 28 E. 29
- 25) For what number k is the circle given by

$$x^2 + 8x + y^2 - 4y = k$$

tangent to the y -axis?

- A. -16 B. -11 C. -4 D. -5 E. 6