40th ANNUAL DR. SUBHASH C. SAXENA MATH CONTEST

March 8, 2019

COASTAL CAROLINA UNIVERSITY

Notes and directions.

- * Do not turn this page over until you are told to do so.
- * Fill in the SCANTRON form according to your proctor's instructions.
- * Calculators are not permitted on this test.
- * You have fifty minutes to complete the test. If you finish early, you should leave quietly and proceed to Hicks Dining Hall for lunch.
- * The test is yours to keep, so use any extra space for scratch work.

Math Contest - Level 1

March 8, 2019

1) If $\frac{52}{x}$ i	s a positive integer, how m	any integer values ar	e possible for x ?	
A. 5	B. 6	C. 7	D. 8	E. 10

2) If $7^9 + 7^9 -$	$+7^9+7^9+7^9+7^9+7^9+7^9$	$=7^x$, what is the va	alue of x ?	
A. 9	B. 10	C. 12	D. 63	E. 9^{7}

3) In a certain election race, all of the 8400 votes were cast for either candidate A or candidate B. If votes for candidate A and votes for candidate B were cast in a 4 to 3 ratio, how many votes were cast for candidate A?

A. 480 B. 840 C. 1200 D. 3600 E. 4800

4) The corners of a square sheet of paper are labeled P, Q, R, and S, as below.



P is folded onto R, and then Q is folded onto R. The area of the resultant figure is 12 square inches. Find the perimeter of the original sheet of paper PQRS.

A. 48 in B. 24 in C. $16\sqrt{3}$ in D. $16\sqrt{2}$ in E. 16 in

5) If $a^{3}b^{4}c^{7} > 0$, which of the following statements must be true? A. ab < 0 B. abc > 0 C. ac > 0 D. ac < 0 E. ab > 0

6) A number x is 32% of a number y. If y is 20% of z, what is z in terms of x? A. 0.064x B. 0.64x C. 6.4x D. $\frac{x}{0.064}$ E. $\frac{x}{0.64}$

7) Two roots of the polynomial $3x^3 + \alpha x^2 - 5x - 10$ are r and -r for some real number r. What is the value of α ?

A. -2 B. 0 C. 3 D. 5 E. 6

8) In a trapezoid RSTV, $\overline{RS} \parallel \overline{VT}$ and $\overline{RV} \perp \overline{VT}$. If $\overline{RS} = \overline{ST} = 13 in$ and $\overline{RV} = 5 in$, find the area of the trapezoid RSTV.

A. $25 in^2$ B. $59 in^2$ C. $65 in^2$ D. $95 in^2$ E. $125 in^2$

9) Tamika selects two different numbers at random from the set {8,9,10} and adds them. Carlos takes two different numbers at random from the set {3,5,6} and multiplies them. What is the probability that Tamika's result is greater than Carlo's result?

A. $\frac{4}{9}$ B. $\frac{5}{9}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$ E. $\frac{2}{3}$

- 10) For what number c is the circle $x^2 8x + y^2 + 4y = c$ tangent to the x-axis?
 - A. -16 B. -11 C. -4 D. -5 E. 6

11) If
$$x \neq -y$$
 and $xy \neq 0$, $\frac{x^{36}-y^{36}}{(x^{18}+y^{18})(x^9+y^9)}$ simplifies to
A. 1 B. $x^9 + y^9$ C. $x^9 - y^9$ D. $x^{18} - y^{18}$ E. $\frac{1}{x^9-y^9}$

- 12) If $\frac{|x+4|}{2} > 5$ and x < 0, which of the following could be the value of x? A. -6 B. -12 C. -14 D. -18 E. 6
- 13) A right pyramid has a hexagonal base with edges of length 16 inches. The lateral edges of the pyramid are 10 inches long. What is the lateral surface area of the pyramid?
 A. 288 in² B. 144 in² C. 288 + 216√3 in² D. 384 in² E. 384 + 216√3 in²
- 14) A $4 \times 4 \times 4$ cubical box sitting on a flat surface contains 64 identical smaller cubes that exactly fill the box. How many of these small cubes touch a vertical side or the bottom of the box?
 - A. 48 B. 52 C. 56 D. 60 E. 64
- 15) The sequence S is defined so that $S_1 = 45$ and $S_n = S_{n-1} + 2$ for each integer $n \ge 2$. What is the sum of the first 100 terms in sequence S?
 - A. 243 B. 14,400 C. 14,500 D. 24,300 E. 24,545

16) What is the 119^{th} letter in the following pattern? ABBCCCDDDD...A. L B. M C. N D. O E. P

17) Consider a correctly set clock that starts ticking at noon. Find the exact angle measure between the minute and the hour hands at 1:15 P.M.

A. 60° B. 57.5° C. 52.5° D. 47.5° E. 42.5°

- 18) Find *a* if $\log_2(\log_3(\log_4(a^3))) = 0$. A. 4 B. 8 C. 16 D. 32 E. 64
- 19) Let the vertices of a parallelogram ABCD be the points A(0,0), B(p,0), C(p+q,r) and D(q,r). Which equation must hold true if the diagonals are perpendicular?

A.
$$\frac{r}{p+q} = -\frac{r}{q-p}$$
 B. $\frac{r}{p+q} = \frac{r}{q-p}$ C. $\frac{r^2}{q^2-p^2} = -1$ D. $\frac{r^2}{q^2-p^2} = 1$ E. $\frac{r}{p+q} = \frac{r}{2q-p}$.

- 20) Three couples have purchased tickets to the school play. Their seats are next to one another in a single row. If each couple sits side by side, how many seating arrangements are possible?
 - A. 8 B. 16 C. 24 D. 48 E. 64.

21) Alex takes a train heading due south. At exactly the same time, his friend Mary who is in a car located 50 miles north of the train starts driving south towards the train on an adjacent roadway parallel to the train track. If the train travels at a constant rate of 50 miles per hour, and the car travels at a constant rate of 80 miles per hour, how long will it take Mary to catch up with Alex?

A. 1 hr B. 1 hr 20 min	C. 1 hr $40 \min$	D. 2 hrs	E. 2 hrs 20 min.
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- 22) A fair coin with heads on one side and tails on the other is tossed three times. What is the probability that the tosses did not include consecutive tails?
 - A. $\frac{3}{8}$ B. $\frac{5}{8}$ C. $\frac{1}{2}$ D. $\frac{3}{4}$ E. $\frac{7}{8}$
- 23) A store is having a 20% off sale today on all items. There is a shirt in the 10% off bin whose original price was \$20. How much does the shirt cost today ?

A. \$14 B. \$14.40 C. \$16 D. \$16.40 E. \$18

- 24) Fiber X cereal is 55% fiber. Fiber Max cereal is 70% fiber. Sheldon combines an amount of the two cereals in a single bowl of mixed cereal that is 65% fiber. If the bowl contains a total of 12 ounces of cereal, how much of the cereal, in ounces, is Fiber X?
 - A. 3 B. 4 C. 6 D. 8 E. 9
- 25) A right triangle has sides of lengths 20 in, 21 in, and 29 in. If a circle is inscribed in this triangle, what is the length of its radius?
 - A. 4 in B. 6 in C. 8 in D. 10 in E. 12 in