

40th ANNUAL DR. SUBHASH C. SAXENA MATH CONTEST

March 8, 2019

COASTAL CAROLINA UNIVERSITY

Notes and directions.

1. Do not turn this page over until you are told to do so.
2. Fill in the SCANTRON form according to your proctor's instructions.
3. Calculators are not permitted on this test.
4. You have 50 minutes to complete the test. If you finish early, you should leave quietly and proceed to Hicks Dining Hall for lunch.
5. The test is yours to keep, so use any extra space for scratch work.

Good luck!

Math Contest Level 2 - March 8, 2019

Coastal Carolina University

1. If the number $2019!$ is written in base 35, how many zeros will it end with?
a) 288 b) 329 c) 334 d) 502 e) other

2. Kevin walks to the All Day, Every Day Music Shop at a rate of 4 mph and returns home, new guitar in hand, at a rate of 3 mph. What was his average rate for the total trip?
a) $24/7$ mph b) 3.5 mph c) $13/4$ mph d) 3.4 mph e) other

3. Three circles of radius 1 are pairwise tangent. A smaller circle is tangent to all three of them. Find the radius of this smaller circle.
a) $\frac{3\sqrt{2}-4}{3}$ b) $\frac{2\sqrt{3}-3}{3}$ c) $\frac{2-\sqrt{3}}{3}$ d) $\frac{3-2\sqrt{2}}{3}$ e) other

4. $1 + 2 + 3 + \cdots + 100 =$
a) 5000 b) 5025 c) 5045 d) 5050 e) other

5. How many solutions of the equation $\log_x(5x - 2) = 3$ are greater than $2/5$?
a) 0 b) 1 c) 2 d) 3 e) other

6. If a circle of radius 4 circumscribes an equilateral triangle, then what is the triangle's perimeter?

- a) $12\sqrt{3}$ b) $12\sqrt{2}$ c) $8\sqrt{3}$ d) 12 e) other

7. Find the smallest prime number that is the fifth term of an increasing arithmetic sequence where the previous four terms are all prime.

- a) 23 b) 29 c) 31 d) 37 e) other

8. Find the minimum value of

$$f(x) = \frac{\sin(x)}{\sqrt{1 - \cos^2(x)}} + \frac{\sec(x)}{\sqrt{1 + \tan^2(x)}} + \frac{\cot(x)}{\sqrt{\csc^2(x) - 1}}.$$

- a) $-2\sqrt{2}$ b) -3 c) $-\sqrt{2}$ d) -1 e) other

9. Suppose that the remainder of a polynomial $p(x)$ when divided by $x - 1$ is 2 and the remainder upon division by $x - 2$ is 1. What is the remainder upon division by the product, $(x - 1)(x - 2)$?

- a) $x - 2$ b) -4 c) $x - 3$ d) 11 e) other

10. Two circles of radius 1 have centers 1 unit apart. Find the area of their intersection.

- a) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ b) $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ c) $\frac{2\pi - \sqrt{3}}{3}$ d) $\frac{\pi + \sqrt{3}}{3}$ e) other

11. What is the coefficient of $x^{10}y^{10}$ in the expansion of $(2x - 3y)^{20}$?
- a) $6^{10} \binom{20}{10}$ b) $-6^{10} \binom{20}{10}$ c) $5^{10} \binom{20}{10}$ d) $-5^{10} \binom{20}{10}$ e) other

12. Find one real solution x of the equation

$$(x + 2)^{35} + (x + 2)^{34}(x - 1) + (x + 2)^{33}(x - 1)^2 + \cdots + (x - 1)^{35} = 0.$$

- a) $-\sqrt{2}/2$ b) $-\sqrt{3}/3$ c) $-1/2$ d) $-1/3$ e) other
13. Let $X = (x, 0)$ be a point on the x-axis. What is the maximum difference between the distances PX and QX where P is the point (2, 10) and Q is the point (14, -5)?
- a) 13 b) 14 c) $10\sqrt{2}$ d) $10\sqrt{3}$ e) other

14. Find the value of $\cos(y) \sin(x)$ if $\sin(x + y) = 0.3$ and $\sin(x - y) = 0.5$.
- a) -0.1 b) -0.2 c) 0.3 d) 0.8 e) other

15. What is the largest integer n such that $20!$ is divisible by 80^n ?
- a) 3 b) 4 c) 5 d) 6 e) other

21. Two cards are in a box. One card is blue on both sides and the other card is red on one side and blue on the other. A card is selected from the box at random and a blue side is observed. What is the probability that the other side is also blue?

- a) $1/4$ b) $1/3$ c) $1/2$ d) $2/3$ e) $3/4$

22. Find the number of integer solutions of the equation $|2x - 5| + |2x + 3| = 8$.

- a) 2 b) 3 c) 4 d) 5 e) other

23. How many functions are there from A to A with $f(f(x)) = 1$ where $A = \{1, 2, 3, 4\}$?

- a) 41 b) 10 c) 24 d) 12 e) other

24. Suppose $\cos(\theta) = \tan(\theta)$. Find the value of $\csc(\theta) + \cos^4(\theta)$.

- a) -1 b) 0 c) 1 d) 2 e) other

25. Let x and y be real numbers. What is the minimum value of the following function?

$$f(x, y) = \sqrt{4 + y^2} + \sqrt{(x - 2)^2 + (2 - y)^2} + \sqrt{(4 - x)^2 + 1}$$

- a) 4 b) $9/2$ c) 5 d) $11/2$ e) other