Math Contest Level 2 - March 2, 2020

Coastal Carolina University

1. The number of zeros at the end of the number $2020!$ if it is written in base 20 is:

a) 502 b) 503 c) 1005 d) 1006 e) other

2. A triangle has a vertex A at $(0,3)$, vertex B at $(4,0)$, and vertex C at $(x,5)$ for some x between 0 and 4. If the area of the triangle is 8, then what is x?

a) $\frac{8}{3}$ b) $\frac{9}{4}$ c) $\frac{\sqrt{10}}{3}$ d) $\frac{\sqrt{15}}{4}$ e) other

3. How many (positive) perfect cubes divide evenly into 60^{60}?

a) $2^7 \cdot 5^3$ b) $3 \cdot 13 \cdot 19^2$ c) $2^3 \cdot 7^2 \cdot 39$ d) $3^2 \cdot 7^2 \cdot 41$ e) other

4. Mark and Mary are running in opposite directions around an oval track and they pass each other every 30 seconds. If Mark runs one lap every 80 seconds, how long does it take Mary to run one lap?

a) 40 sec b) 45 sec c) 48 sec d) 50 sec e) other

5. Consider all ordered pairs (x, y) that satisfy the equations $x^2 + y^2 = xy + 13$ and $x + y = 2xy - 17$. The largest possible sum of the ordered pair is:

a) 1 b) 3 c) 5 d) 7 e) other

6. What is the largest number that can not be achieved by adding positive integer multiples of 6, 9, and 20?

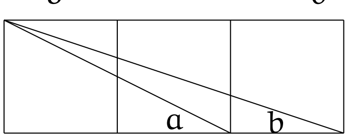
a) 28 b) 37 c) 43 d) 51 e) other

7. How many integer pairs solve $x^2 + 6x + y^2 = 16$?

a) 6 b) 8 c) 10 d) 12 e) other

8. The following image made up of 3 equal squares. Find $a + b$ (in radians).

a) $\frac{\pi}{8}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$ e) other



9. Let $f(x, y)$ be defined by, $f(x, 0) = x$ and $f(x, y + 1) = f(f(x, y), y)$. Which of the following is the largest?

a) $f(10, 15)$ b) $f(11, 14)$ c) $f(12, 13)$ d) $f(13, 12)$ e) $f(14, 11)$

10. Find the value of x that makes the sequence geometric: $x,\ 3x + 1,\ 6x + 2,\ \ldots$

a) -1 b) 0 c) 1 d) 2 e) other

11. Let x and y be randomly chosen numbers between 0 and 1. What is the probability that $\dfrac{y - x}{y + x}$ is closest to an odd integer?

a) 1/4 b) 1/3 c) 2/5 d) 1/2 e) other

12. Circles of radius 10 and 17 intersect at two points. The chord made by these two points has length 16. The sum of the possible values for the distance between their centers is:

a) 27 b) 28 c) 29 d) 30 e) other

13. What is the number of positive integer solution pairs to the equation $2x + 3y = 515$?

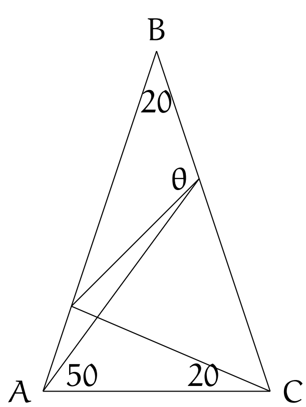
a) 85 b) 86 c) 170 d) 171 e) other

14. For $n \geq 3$, let $f(n) = \log_2(3) \cdot \log_3(4) \cdots \log_{n-1}(n)$. What is the value of $\sum_{k=2}^{99} f\left(2^k\right)$?

a) 1288 b) 4949 c) 2992 d) 2048 e) other

15. Find θ given that ABC is an isosceles triangle (all angles are in degrees).

a) 100 b) 110 c) 120 d) 130 e) other



16. Two roots of the polynomial $3x^3 + \alpha x^2 - 5x - 10$ are r and $-r$ for some real number r. What is the value of α?

a) -6 b) -2 c) 3 d) 6 e) other

17. If x and y are real numbers with $x + y = 2$ and $x^4 + y^4 = 1234$ then $xy =$

a) -21 b) -22 c) -23 d) -24 e) other

18. Find $f(2)$ if $f\left(1 - \frac{1}{x}\right) + 2f\left(\frac{1}{1-x}\right) + 3f(x) = x^2$ for $x \neq 0, 1$.

a) 31/24 b) 23/34 c) 19/37 d) 16/41 e) other

19. Find the number of solutions to the system $\sin(x + y) = 0$, $sin(x - y) = 0$ with $0 \leq x \leq \pi$ and $0 \leq y \leq \pi$.

a) 4 b) 5 c) 6 d) 9 e) other

20. A point P is chosen inside of the square ABCD. What is the probability that the angle APB is obtuse?

a) $\pi/12$ b) 1/4 c) $\pi/8$ d) 1/2 e) other

21. Given that $\log_5(\sin x) + \log_5(\cos x) = -1$, find $|\sin^2 x \cos x + \cos^2 x \sin x|$.

a) $\frac{1}{5}$ b) $\frac{13}{25}$ c) $\frac{2}{5}$ d) $\frac{6}{25}$ e) other

22. What is the the sum of all possible integers n such that $n^2 + 2n + 2$ divides evenly into $n^3 + 4n^2 + 4n - 14$?

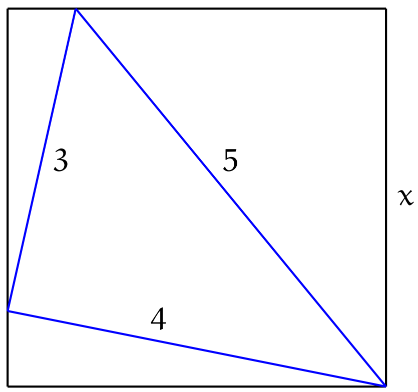
a) -11 b) -2 c) 0 d) 1 e) other

23. Consider the points $P(2, 0)$, $Q(-1, 4)$ and $R(0, y)$ in the xy-plane. Find y such that $|PR - QR|$ is maximized.

a) 0 b) 8/3 c) 4 d) 8 e) other

24. A triangle with sides 3,4 and 5 is inscribed a square. Find the side length of the square.

a) $\frac{15}{\sqrt{19}}$ b) $\frac{15}{\sqrt{17}}$ c) $\frac{16}{\sqrt{19}}$ d) $\frac{16}{\sqrt{17}}$ e) other



25. Two players are playing a game involving two pieces and an octahedron. The pieces are initially on opposite vertices of the octahedron and move along the edges from one vertex to an adjacent vertex. The players alternate turns and move only their piece on their turn. A player is declared the winner if he moves his piece onto the vertex occupied by his opponents piece. If the players make their moves randomly, what is the probability that the player who moves first wins?

a) 2/7 b) 1/3 c) 2/5 d) 3/7 e) other