

42nd ANNUAL DR. SUBHASH C. SAXENA MATH CONTEST

Level 2

March 3, 2023

COASTAL CAROLINA UNIVERSITY

Notes and directions.

1. Do not turn this page over until you are told to do so.
2. Fill in the SCANTRON form according to your proctor's instructions.
3. Calculators are not permitted on this test.
4. You have 50 minutes to complete the test. If you finish early, you should leave quietly and proceed to Hicks Dining Hall for lunch.
5. The test is yours to keep, so use any extra space for scratch work.

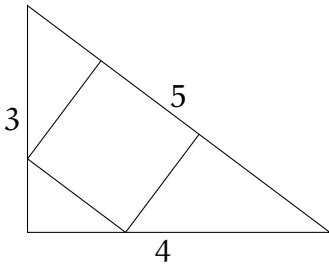
Good luck!

Math Contest Level 2

1. Find the number of zeros at the end of the number $2023!$ if it is written in base 11.
a) 199 b) 200 c) 201 d) 202 e) other

2. Two fair, six-sided dice are rolled repeatedly until either a sum of 5 or 7 appears. Find the probability that the sum of 5 appears first.
a) $2/5$ b) $2/3$ c) $2/7$ d) $5/18$ e) other

3. Find the area of the square inscribed in a 3-4-5 triangle.
a) $\sqrt{60}/37$ b) $60/37^2$ c) $60^2/37$ d) $60^2/37^2$ e) other



4. Given that $5^x = 7^y = 1225$, find $\frac{xy}{x+y}$
a) 2 b) 3 c) 4 d) 5 e) other

5. How many ordered pairs (x, y) of integers satisfy $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$?
- a) 5 b) 7 c) 9 d) 11 e) other

6. The expression $\frac{\cot^2(\theta) - \tan^2(\theta)}{2 + \cot^2(\theta) + \tan^2(\theta)}$ is equal to which of the following?
- a) $\cos(\theta)$ b) $\cos(2\theta)$ c) $\sin(\theta)$ d) $\sin(2\theta)$ e) other

7. A piece of string is wrapped around a rod four times. The string starts at the bottom and ends at the top and has uniform spacing as it is wrapped. If the rod has a circumference of 4 cm and a length of 12 cm, how long is the string?
- a) 20 b) $20\sqrt{2}$ c) 20π d) $20\pi\sqrt{2}$ e) other

8. If $x > 0, x \neq 1$, and $(\log_2 x)^2 = \log_4 x$, then:
- a) $1 < x \leq 1.25$ b) $1.25 < x \leq 1.5$ c) $1.5 < x \leq 1.75$ d) $1.75 < x \leq 2$ e) other

9. At 3:00, the hour and minute hands of a clock are at a right angle. To the nearest minute, what is the next time the hour and minute hands of that clock are at a right angle?

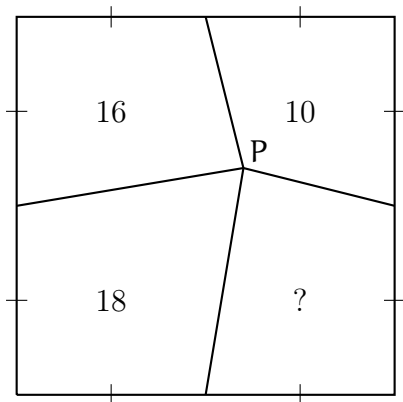
- a) 3:31 b) 3:32 c) 3:33 d) 3:34 e) other

10. Suppose $\frac{a^2 - bc}{a^2 + bc} + \frac{b^2 - ca}{b^2 + ca} + \frac{c^2 - ab}{c^2 + ab} = 1$. Find $\frac{a^2}{a^2 + bc} + \frac{b^2}{b^2 + ca} + \frac{c^2}{c^2 + ab}$.

- a) 2 b) 2.5 c) 3 d) 3.5 e) other

11. A point P is drawn inside a square and lines are drawn from the midpoints of each side of the square to the point P. Given the area of three of the sections, find the area of the 4th section.

- a) 11 b) 12 c) 13 d) 14 e) other



12. The sum of 2023 consecutive integers is 20230. Find the greatest of these integers.

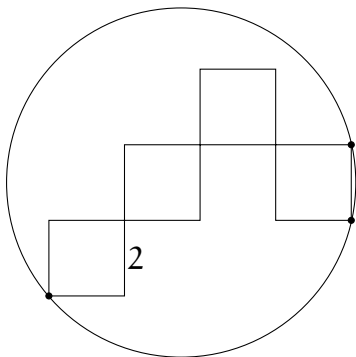
- a) 1020 b) 1021 c) 1022 d) 1023 e) other

13. Suppose x is in radians with $0 \leq x \leq \pi$. Find the sum of the solutions to the equation $81^{\sin^2(x)} + 81^{\cos^2(x)} = 30$
- a) $\pi/2$ b) π c) $3\pi/2$ d) 2π e) other

14. How many paths of six moves or less may a King follow on a 5-by-5 chessboard in getting from the bottom left square to the top right square without backtracking? The possible moves are 1 space to the right, 1 space up, or 1 space diagonally up and to the right.
- a) 90 b) 91 c) 110 d) 111 e) other

15. Find the sum of the solutions to $\frac{27^x + 343^x}{63^x + 147^x} = \frac{37}{21}$
- a) $\frac{58}{21}$ b) -1 c) 0 d) 1 e) other

16. Find the radius of the following circle, given that each square has a side length of 2.
- a) $\sqrt{85}$ b) $\sqrt{85}/2$ c) $\frac{\sqrt{85}}{3}$ d) $2\sqrt{85}$ e) other



17. Given that $x\sqrt{y} + y\sqrt{x} = 182$ and $x\sqrt{x} + y\sqrt{y} = 183$, find $x + y$.
a) $9/365$ b) $365/9$ c) 365 d) $1/365$ e) other

18. Find the sum: $\sin^4(1^\circ) + \cos^4(1^\circ) + \dots + \sin^4(45^\circ) + \cos^4(45^\circ)$
a) $65/2$ b) $65/3$ c) $69/2$ d) 23 e) other

19. A point P is chosen inside an equilateral triangle of side length 1, and perpendiculars drawn to the three sides. Let $S(P)$ denote the sum of the lengths of the three perpendiculars. What is the largest possible value for $S(P)$ out of all points P inside the triangle?
a) $\sqrt{3}$ b) $\sqrt{2}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{\sqrt{3}}{3}$ e) other

20. What is the max value of $(\log(xy))^2$, given $(\log(x))^2 + (\log(y))^2 = \log(x^2) + \log(y^2)$?
a) 12 b) 14 c) 16 d) 18 e) other

24. Solve for x : $x^3 = \frac{6}{\log_3(x)}$

a) $\sqrt[3]{3}$

b) $\sqrt[3]{6}$

c) $2\sqrt[3]{3}$

d) $2\sqrt[3]{6}$

e) other

25. The semi-circle shown is folded along the dotted line. The curved segment touches the diameter, dividing it into lengths of 4 and 2. What is the length of the dotted line?

a) $2\sqrt{6}$

b) 5

c) $\sqrt{26}$

d) $3\sqrt{3}$

e) other

