Math Contest–Level 1
March 24, 2006

1. a, b are distinct digits from the set $\{2, 3, 4, 5, 8, 9\}$, such that this array is a correct multiplication.

		7	a	4
	x		b	a
	b	b	0	b
1	4	6	8	
1	6	8	8	b

The value of a and b are respectively,

(a) $a = 2, b = 2$ (b) $a = 7, b = 2$ (c) $a = 3, b = 1$ (d) $a = 3, b = 2$
(e) None of the above

2. Two squares are inscribed in a semicircle as shown. If the area of the smaller square is 25, then the area of the larger square is

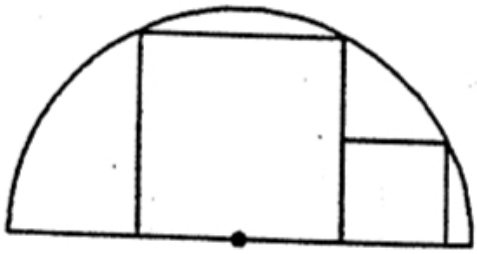
(a) 80

(b) 100

(c) 120

(d) 60

(e) none of the above



3. What is the integer n for which $5^n + 5^n + 5^n + 5^n + 5^n = 5^{25}$?

(a) 23 (b) 1 (c) 24 (d) 25 (e) None of the above

4. Use the numbers 25,26,35,45,46 (using each number exactly once) to replace the letters a, b, c, d, e so that each of the digits 1,2,3,4,5 and 6 appears exactly once in each of the five columns.

12	13	14	15	16
34	a	b	23	24
56	c	36	d	e

The right sequence of the letters a, b, c, d, e (in that order) is

(a) 26,35,45,25,46 (b) 46,45,25,35,26
(c) 26,25,45,46,35 (d) 26,25,46,45,35

5. Let $N = 123456789$. If the digits 1 and 4 are interchanged in N, we obtain 423156789, which is an integral multiple of 11. How many integral multiples of 11 that can be obtained by interchanging two digits of N?

(a) 1 (b) 2 (c) 3 (d) 4 (e) more than 4

6. Let $A = 361 + 362 + 363 + \ldots + 726$, that is, let A be the sum of the consecutive positive integers beginning with 361 and ending with 726. Also, let

$$\text{B} = 363{+}364{+}\ldots{+}727, \quad \text{C} = 365{+}366{+}\ldots{+}728,$$
$$\text{D} = 367{+}368{+}\ldots{+}729, \quad \text{E} = 369{+}370{+}\ldots{+}730.$$

The smallest and largest of the five sums A,B,C,D,E are respectively,

(a) A,D (b) E,D (c) E,C (d) A,C (e) none of the above

7. The diagonals of a rhombus measure 24 inches and 32 inches. The perimeter of the rhombus

   (a) cannot be determined uniquely from the given information.

   (b) must be 80 inches.

   (c) must be $80\sqrt{2}$ inches.

   (d) must be 160 inches.

   (e) must be $160\sqrt{2}$ inches.

8. A,B and C are the vertices of a triangle whose perimeter is 73 inches. AB is the longest side of the triangle, and the product of the lengths of the sides AB and BC is 750 square inches. Side BC is 7 inches longer than side AC. The length of side AB

   (a) cannot be determined uniquely from the given information.

   (b) must be 15 inches.

   (c) must be 25 inches.

   (d) must be 30 inches.

   (e) must be 50 inches.

9. C_1 and C_2 are concentric circles with radii r_1 and r_2, respectively, with $r_1 < r_2$. Points A and B are chosen on C_2 in such a way that the chord AB is tangent to C_1 and has length 16 centimeters. If A is the area of the annulus (the ring shaped region) bounded by C_1 and C_2

   (a) A cannot be determined uniquely from the given information.

   (b) $A < 200$.

   (c) $200 \leq A < 500$.

   (d) $500 \leq A < 800$.

   (e) $A > 800$

10. The value of $(5^{-1} - 6^{-1})^{-1}$ expressed as a rational number is

    (a) $\frac{1}{30}$ (b) 30 (c) $\frac{1}{24}$ (d) 24 (e) none of the above

11. A line of people has gathered in front of a theater six hours before a matinee showing of the sci-fi adventure, *The Menacing Phantom*. The ration of females to males in the line is 6:7. After 15 minutes, another 21 people have joined the line and the ratio of females to males has become 7:8. The fewest number of people that could have originally been in the line is

    (a) 39 (b) 26 (c) 30 (d) 42 (e) none of the above

12. ABCD is a rectangle. Points E and F are chosen as follows. Segment BA is extended through A to a point E, and F is the intersection of segment EC with edge AD. Suppose that the measure of angle ACD is 60 degrees and that the length of the segment EF is twice the length of the diagonal AC. To the nearest degree, the measure of angle ECD is

    (a) 70 (b) 30 (c) 60 (d) 20 (e) none of the above

13. A full tank of propane weighs 32 pounds. After $\frac{3}{4}$ of the tank is used up, the tank and the remaining gas weigh 14 pounds. What is the weight of the tank when empty?

    (a) 5 (b) 7 (c) 24 (d) 8 (e) none of the above

14. In triangle ABC, angle ACB is 50 degrees and angle CBA is 70 degrees. Let D be the foot of the perpendicular from A to BC, O the center of the circle circumscribed about triangle ABC and E the point on this circle which is diametrically opposite A. The angle DAE in degrees is

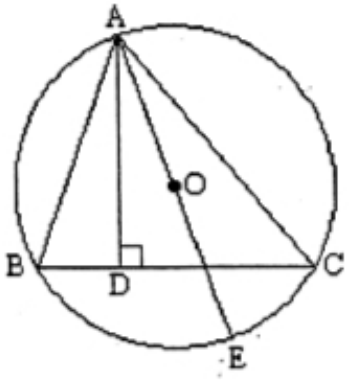
    (a) 40

    (b) 100

    (c) 20

    (d) 140

    (e) none of the above



15. Let D be any point on the base of isoceles triangle ABC. Extend AC to E so that CD = CE. Extend ED to meet AB at F. If angle CED is 10 degrees, find angle AFD in degrees.

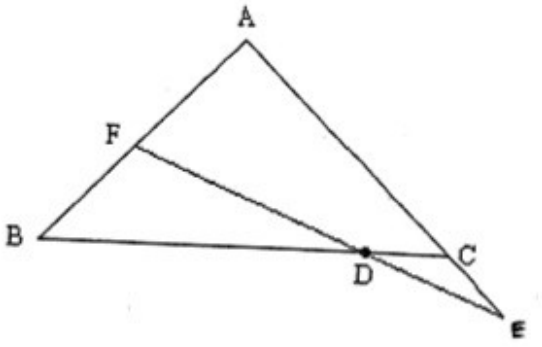
(a) 40

(b) 30

(c) 20

(d) 10

(e) none of the above



16. The solution $(x+2)(x-2)(x-13) = (x+2)(x-7)(x-11)$ is

(a) 3 (b) 2 (c) -17 (d) -2 (e) none of the above

17. If a positive number x satisfies $x^2 + \dfrac{1}{x^2} = 4$, the value of $x + \dfrac{1}{x}$ is

(a) 6 (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{6}$ (e) none of the above

18. In the figure, E is the midpoint of AB and F is the midpoint of AD. If the area of FAEC is 13 square units, what is the area of ABCD?

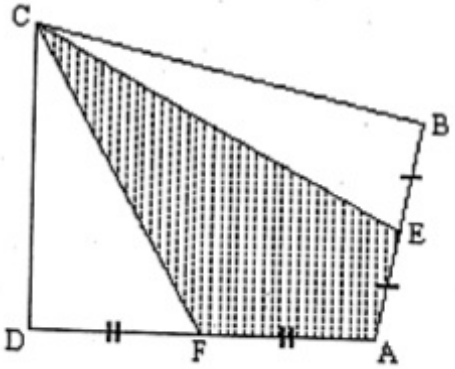
(a) 26

(b) 39

(c) 42

(d) 20

(e) none of the above



19. In the figure, AC is 39 units, AD is x units, DB is 9 units, DE is 13 units, BE is x units and $\angle$ BDE = $\angle$ BCA. What is the value of x?

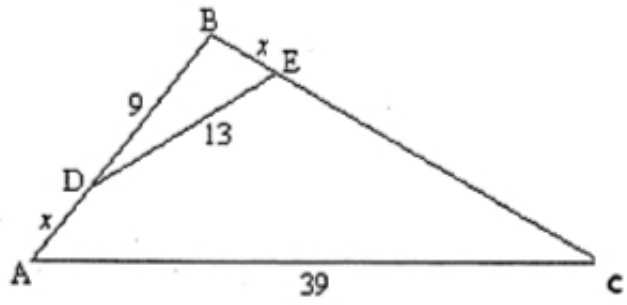
(a) 4.5

(b) 9

(c) 2/9

(d) 13

(e) none of the above



20. In how many ways can the letters A,B,C and D be arranged in a sequence so that A is not in position 3, B is not in position 1, C is not in position 2 and D is not in position 4?

(a) 6 (b) 3 (c) 9 (d) 12 (e) none of the above

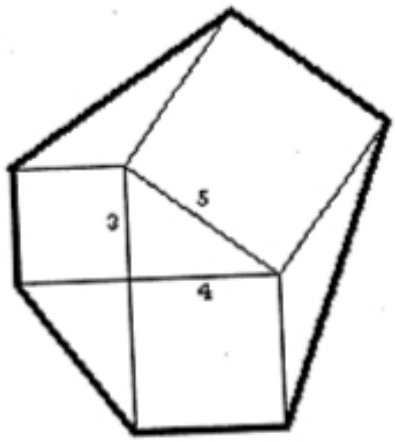
21. In right triangle ABC, where the right angle is at vertex C, the altitude from vertex C divides the hypotenuse into two segments, one of length 16 units and the other of length 1 unit. Find the area of the triangle.

(a) 16 (b) 34 (c) 32 (d) 2 (e) none of the above

22. Anna and her brother Bob decide to ride their bikes to the candy store, each taking the same route. Anna leaves home at 3:00 pm and rides at a constant speed of 8 mph until she reaches the store. She spends 5 minutes and 30 seconds at the store, then heads home riding at a constant speed of 5 mph. At 3:25 pm, Anna meets Bob, who left the house at 3:12 pm and is riding at a constant speed of 6 mph. How far (in miles) is Anna and Bob's house from the store?

(a) 9 (b) 5 (c) 9/5 (d) 5/9 (e) none of the above

23. Given a right triangle with sides of length 3,4 and 5 units, construct a square on each side of the triangle. Then construct a hexagon by joining the vertices of the squares which are not vertices of the triangle. See the figure. The area of the hexagon is



(a) 68

(b) 50

(c) 62

(d) 74

(e) none of the above

24. A certain 90-mile trip took 2 hours. Exactly $\frac{1}{3}$ of the distance traveled was by rail, and this part of the trip took $\frac{1}{5}$ of the travel time. What was the average rate, in miles per hour of the rail portion of the trip?

(a) 12 mph (b) 30 mph (c) 45 mph (d) 60 mph (e) none of the above

25. In triangle ABC, BC =13, CA = 14, and AB = 15. If D is a point on $\bar{CA}$ such that $\bar{BD}$ is perpendicular to $\bar{CA}$, then what is BD?

(a) 9

(b) 10

(c) 11

(d) 12

(e) none of the above