

Math Contest Level 1 - March 27th, 2009

Coastal Carolina University

- How many of the numbers 555555, 5555555, 55555555, 555555555 and 5555555555 are divisible by 9?
(a)0 (b)1 (c)2 (d)3 (e)none of these
- A quadratic polynomial $p(x)$ satisfies $p(0) = 3$, $p(1) = 5$, $p(2) = 8$. Then $p(5)$ is
(a) 21 (b) $21\frac{1}{2}$ (c)22 (d) $22\frac{1}{4}$ (e) none of these
- There are 8 students in David's Linear and Discrete Mathematics class. Knowing there are 3 students who major in electrical engineering, 4 students who major in mathematics, and 1 student who double majors in both electrical engineering and mathematics, how many students in Davids class major in neither electrical engineering nor mathematics?
(a) 0 (b) 1 (c) 2 (d) 3 (e) 4
- Four consecutive odd integers are added. If the smallest one is $2m-1$, then the sum equals
(a) $8m-10$ (b) $8m+2$ (c) $8m+8$ (d) $8m+10$ (e) $8m +3$
- Find the value of $\sqrt{5 + 2\sqrt{6}} - \sqrt{5 - 2\sqrt{6}}$
(a) $4\sqrt{6}$ (b) $2\sqrt{2}$ (c) $2\sqrt{3}$ (d) $\frac{3\sqrt{3}}{2}$ (e) none of these

6. Pizzas are to be ordered for a party. The restaurant offers three specials.
Special A: a small, together with a medium pizza, for 13 dollars.
Special B: 2 medium pizzas for 14 dollars.
Special C: a large pizza for 15 dollars.
A small pizza has a diameter of 12 inches, a medium pizza 16 inches and a large pizza 20 inches. Rank A, B and C from the greatest to the least square inches per dollar.
(a) CBA (b) BCA (c) ABC (d)ACB (e) BAC
7. Several roosters want to buy an alarm clock. If each contributes \$0.35, they lack \$4.40. If each contributes \$0.40, they have \$4.40 extra. The number of roosters is in the range of
(a) less than 50 (b) 50 to 100 (c) 100 to 150 (d) 150 to 200 (e) more than 200
8. Triangle ABC has vertices (0, 0), (11, 60), and (91, 0); respectively. The line $y = kx$ cuts the triangle into two triangles of equal area. Find k.
(a) $\frac{4}{3}$ (b) $\frac{25}{37}$ (c) $\frac{3}{5}$ (d) $\frac{4}{7}$ (e) $\frac{30}{51}$
9. Which of the following is greatest?
(a) 6^{24} (b) 9^{18} (c) 11^9 (d) $(2.8)^{35}$ (e) $(2.8)^{33}$
10. How many different chords are determined by 8 distinct points lying on a circle ?
(a) 24 (b)26 (c) 28 (d) 30 (e) 32

11. A unicorn is buying rutabaga to feed his young unicorn children, of which he has 6. Child A eats 17 pounds of food every 13 days. Child B eats 3 pounds of food every 2 days. Child C eats a pound of food per day. Child D eats no food, ever, and instead survives on dewdrops. Child E eats 10 pounds of food every 8 days. Child F eats 10 pounds of food every 6.5 days. If he begins feeding his children on day 1, on what day will the 687th pound of food be eaten?
(a) 105 (b) 169 (c) 104 (d) 338 (e) none of these
12. A fair die is rolled six times. If p is the probability that each of the six faces on the die appears exactly once among the six rolls, which of the following statements about p is correct?
(a) $p < 0.02$ (b) $0.02 < p < 0.04$ (c) $0.04 < p < 0.08$ (d) $0.08 < p < 0.16$ (e) $0.16 < p$
13. A sphere is inscribed in a cone whose radius is 6 in. and whose height is 8 in. What is the volume of the sphere?
(a) $\frac{32}{3}\pi \text{ in}^3$ (b) $36\pi \text{ in}^3$ (c) $27\pi \text{ in}^3$ (d) $9\pi \text{ in}^3$ (e) $\frac{4\sqrt{3}}{3}\pi \text{ in}^3$
14. If n is the number of integers between 1 and 999 that have at least one digit equal to seven in their base ten representation, then
(a) $100 < n \leq 150$ (b) $150 < n \leq 200$ (c) $200 < n \leq 250$ (d) $250 < n \leq 300$
(e) $300 < n \leq 350$
15. The lower two vertices of a square lie on the x-axis and the upper two vertices of the square lie on the parabola $y = 15 - x^2$. What is the area of the square?
(a) 9 (b) $10\sqrt{2}$ (c) 16 (d) 25 (e) 36

16. Let $A = 2^0 + 2^1 + \dots + 2^{1000}$. If you divide A by 15, what is the remainder?
(a) 0 (b) 1 (c) 2 (d) 3 (e) 4
17. What digit occupies the 38,885th position when we write out all the integers in succession, beginning with 1 (i.e. 1234567891011121314151617...)?
(a) 1 (b) 3 (c) 5 (d) 8 (e) 9
18. What is the coefficient of x^3 of the expansion of $(1 + x + x^2 + x^3 + x^4 + x^5 + x^6)^6$?
(a) 40 (b) 46 (c) 48 (d) 56 (e) 62
19. A circular table is pushed into a corner in a rectangular room so that it touches both walls. A point on the edge of the table between the two points of contact is 2 inches from one wall and 9 inches from the other wall. What is the radius of the table?
(a) 5 (b) 12 (c) 15 (d) 17 (e) 20
20. A friend has a stack of ten cards numbered 1 through 10. He shuffles the cards and you draw two cards. What is the probability that the cards are consecutive numbers??
(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{2}{9}$ (e) $\frac{3}{16}$

21. Suppose toothpicks of length 2 inches are aligned point-to-point on a desk in the shape of a closed rectangle. What is the least area (in inches²) that can be enclosed by this rectangle if it has a perimeter of 40 inches ?
 (a) 9 (b) 400 (c) 100 (d) 36 (e) none of these
22. Find the number of distinguishable permutations of the letters in the word:
 'coastalcarolina' ?
 (a) 10^{15} (b) $\frac{15!}{10!5!}$ (c) $\frac{15!}{(2!)^3 4!}$ (d) $15!$ (e) none of these
23. A class of 10 students took a math test. Each problem was solved by exactly 7 of the students. If the first nine students each solved 4 problems, how many problems did the tenth student solve?
 (a) 2 (b) 3 (c) 4 (d) 5 (e) 6
24. What are the last two digits of 103^{4205} ?
 (a) 01 (b) 23 (c) 43 (d) 63 (e) none of these
25. Calculate the expression: $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + 100 \cdot 100!$
 (a) $113 \cdot 100! + 1$ (b) $101! - 1$ (c) $102! - 100! + 13$ (d) $(100!)^2 - 103$
 (e) $2 \cdot 100! - 100^3 + 17$