COASTAL CAROLINA UNIVERSITY

March 8, 2013

The 34th Annual Dr. Subhash Saxena Math Contest

LEVEL 1

Notes and Directions:

- Do not turn this page until you are told to do so.
- Fill in the SCANTRON form according to your proctor's instructions. Make sure you put your name and your school's name at the top.
- Calculators are not permitted on this test.
- You have 50 minutes to complete the test. If you finish early, leave the classroom quietly and proceed to the Hicks Dining Hall for lunch.
- The test is yours to keep. Use any extra space for scratch work.

Math Contest – Level 1
March 8, 2013

1) Order $-4.34, -4.3\overline{4}, -4.\overline{34}, -4.3\overline{43}, -4.\overline{434}$ from least to greatest.

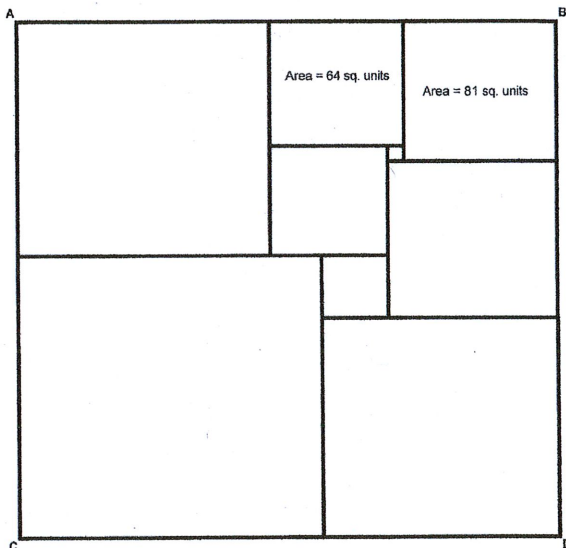   A) $-4.\overline{434} < -4.3\overline{4} < -4.3\overline{43} < -4.\overline{34} < -4.34$

   B) $-4.34 < -4.\overline{34} = -4.3\overline{43} < -4.3\overline{4} < -4.\overline{434}$

   C) $-4.3\overline{4} < -4.3\overline{43} = -4.\overline{34} < -4.34 < -4.\overline{434}$

   D) $-4.\overline{34} < -4.3\overline{43} < -4.34 < -4.3\overline{4} < -4.\overline{434}$

   E) $-4.\overline{434} < -4.3\overline{4} < -4.3\overline{43} = -4.\overline{34} < -4.34$

2) Quadrilateral *ABCD* has been dissected into squares. The area of the square in the upper right corner is 81 sq. units. The area of the square to the left of that square is 64 sq. units. What is the area of the quadrilateral *ABCD*?

   A) 1055 sq. units   B) 1156 sq. units   C) 1056 sq. units
   D) 1024 sq. units   E) 1049 sq. units

3) How many different amounts of money can you have if you have coins consisting of quarters, dimes, and nickels and the total number of coins you have is 4? (You don't have to have some of each coin. You could have, for example, 4 quarters, no dimes, and no nickels.)

A) 14   B) 15   C) 13   D) 11   E) 12

4) A fruit grower packs apples in boxes with each box containing at least 240 apples and at most 250 apples. How many boxes of apples must be selected to be certain that at least three boxes contain the same number of apples?

A) 23   B) 30   C) 13   D) 21   E) 33

5) How many squares are in the following figure?
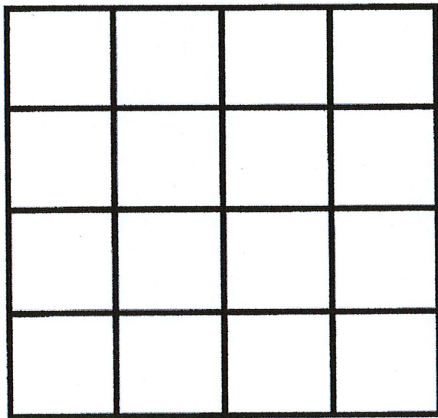
A) 16   B) 17   C) 29   D) 30   E) 31

6) How many times in a mother's life will she be exactly twice the age of her only son? (Assume a life expectancy of 85 years.)

A) 0   B) 1   C) 2   D) 4   E) Depends on how long the mother lives.

7) In a stock car race, the first five finishers were a Ford, Pontiac, Chevrolet, Dodge, and a Toyota (not necessarily in that order). If the following is known, in what order, from, 1st to 5th, did the cars finish the race?

The Ford finished 7 seconds before the Chevrolet. The Pontiac finished 6 seconds after the Toyota. The Dodge finished eight seconds after the Toyota. The Chevrolet finished 2 seconds before the Pontiac.

   A) Toyota, Ford, Chevrolet, Pontiac, and Dodge
   B) Toyota, Ford, Dodge, Chevrolet, and Pontiac
   C) Ford, Toyota, Chevrolet, Pontiac, and Dodge
   D) Toyota, Ford, Dodge, Pontiac, Chevrolet
   E) Ford, Chevrolet, Toyota, Pontiac, Dodge

8) Abby walks from town A to town B at a constant speed of 3 mph, and back from town B to town A at a constant speed of 5 mph. What is Abby's speed on the round-trip walk?

   A) 3.5 mph   B) 3.75 mph   C) 4 mph   D) 4.25 mph   D) 4.5 mph

9) What is the 2013th digit in the decimal expansion of $\dfrac{3}{7}$?

   A) 1   B) 2   C) 8   D) 5   E) 7

10) Four friends go fishing one day and bring home a total of 11 fish. If each person caught at least one fish, which of the following must be true?

   A) One person caught exactly 2 fish.
   B) One person caught exactly 3 fish.
   C) One person caught more than 3 fish.
   D) Two people each caught more than 1 fish.
   E) One person caught fewer than 3 fish.

11) Assume that the following pattern of square tile figures continues. If the number of tiles in a_n is 239, find n.

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad , \ldots$
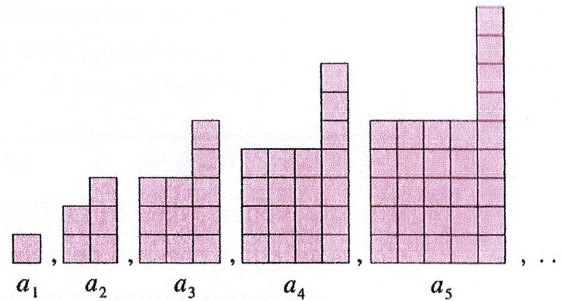
A) 13   B) 14   C) 15   D) 16   E) 17

12) A city council is composed of 6 men and 5 women. Four members are to be chosen as delegates. In how many ways can exactly 2 men and 2 women be chosen?

A) 30   B) 6   C) 11   D) 55   E) 150

13) What is the correct ordering of the numbers 2^{48}, 5^{24}, and 10^{16}?

A) $10^{16} < 2^{48} < 5^{24}$ B) $2^{48} < 5^{24} < 10^{16}$ C) $5^{24} < 2^{48} < 10^{16}$

D) $2^{48} < 10^{16} < 5^{24}$ E) $10^{16} < 5^{24} < 2^{48}$

14) John took all of his money out of his savings account. He spent \$50 on a radio and $\dfrac{3}{5}$ of what remained on presents. Half of what was left he put back in his savings account and the remaining \$35 he donated to charity. How much did John originally have in his savings account?

A) \$312.50   B) \$190.00   C) \$187.50   D) \$300.00   E) \$225.00

15) Find an algebraic expression in terms of q and n that represents the amount of bacteria after n minutes if the initial amount of bacteria, q, triples every 30 seconds.

A) $3^{2n}q$ B) $3q^n$ C) $3q^{2n}$ D) $3^n q$ E) $(3q)^{2n}$

16) The length of the diagonal of a cube is 6. What is the surface area of the cube?

A) $36\sqrt{3}$ B) 72   C) 54   D) 108   E) $54\sqrt{3}$

17) Two circles lying in the same plane have the same center. The radius of the larger circle is twice the radius of the smaller circle. If the area of the region between the two circles is 9, then what is the radius of the smaller circle?

A) $\dfrac{3}{\pi}$ B) $\dfrac{\sqrt{3}}{\pi}$ C) $\dfrac{\sqrt{\pi}}{3}$ D) $\dfrac{\sqrt{\pi}}{3\pi}$ E) $\dfrac{\sqrt{3\pi}}{\pi}$

18) Find the area of the shaded part of the following figure. Assume all arcs are circular with centers marked.

square

10 cm
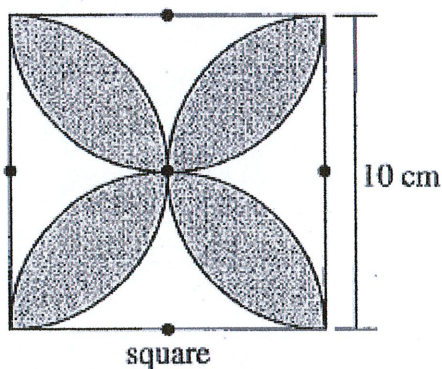
A) $50\pi - 100$ B) $25\pi - 50$ C) $\dfrac{25\pi}{2}$ D) $100 - 25\pi$ E) $100 - 50\pi$

19) What is the next term in the sequence 5 6 14 32 64 115 191

    A) 267   B) 283   C) 244   D) 299   E) 298

20) Let a be a real number. Consider a polynomial function given by
$$f(x) = ax^3 - (a-1)x^2 + ax - (a-1)$$

Which of the following is true?

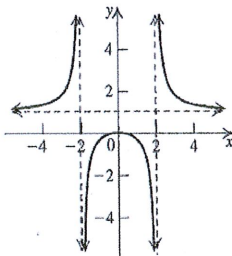I. $x = \dfrac{a-1}{a}$ is a root of f

II. The sum of the roots of f is $\dfrac{a-1}{a}$

III. The product of the roots of f is $\dfrac{a-1}{a}$

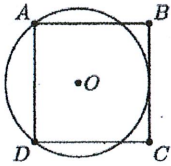IV. As $x \to \infty$, $f(x) \to -\infty$ if $a > 0$

    A) I   B) I and II   C) I, II, and III   D) I, II, III, and IV   E) None of the above

21) Which one of the following functions matches the given graph of f?

    A) $f(x) = \dfrac{x+2}{x-2}$ B) $f(x) = \dfrac{x^3}{(x-2)^2}$ C) $f(x) = \dfrac{2x^2}{x^2-4}$ D) $f(x) = \dfrac{x^2}{x^2-4}$

22) In the figure shown, a circle with center O is drawn through vertices A and D and tangent to side $\overline{BC}$ of square $ABCD$ with side length 8. What is the radius of the circle?

A) 3   B) 5   C) $3\sqrt{2}$ D) $4\sqrt{2}$ E) 6

23) The zeros of the polynomial $x^3 - 33x^2 + 354x + k$ are in arithmetic progression (three consecutive terms in an arithmetic sequence). What is the value of k?

A) - 1232   B) 1430   C) -803   D) -1320   E) – 1210

24) Define $<a,b>$ by $<a,b> \equiv \dfrac{a}{a-b}$. If $<s,t> = 5$, what is $<t,s>$?

A) -7   B) -5   C) -4   D) 4   E) 2

25) If $-1 < x < 0$, then $\left| x - \dfrac{1}{x} \right| + \left| \dfrac{1}{x} + x \right| =$

A) $-2x$ B) $2x$ C) 0   D) $-\dfrac{2}{x}$ E) $\dfrac{2}{x}$