

Math Contest Level 2 - March 7, 2014

Coastal Carolina University

1. A lily pad on a pond doubles in size (area) every day. After 32 days, it covers the pond exactly. How long did it take to cover half of the pond?

- a) 8 days      b) 16 days      c) 31 days      d) 33 days      e) none of these

2. Find the length of the longest diagonal of a rectangular solid with sides of length 8, 9 and 12.

- a) 17      b) 21      c) 25      d) 29      e) none of these

3. If  $16^{x+1} = 3$ , what is  $2^{4x+2}$ ?

- a)  $\frac{3}{4}$       b)  $\frac{3}{2}$       c) 3      d) 6      e) 12

4. Find the exact value of  $\tan(5\pi/12)$ .

- a)  $-1 + \sqrt{3}$       b)  $1 + \sqrt{3}$       c)  $-2 + \sqrt{3}$       d)  $2 + \sqrt{3}$       e) none of these

5. What is the remainder when  $1 + 2 + 3 + \dots + 2014$  is divided by 2014?

- a) 1      b) 3      c) 537      d) 1009      e) none of these

6. If  $n \geq 2$  is a natural number, which of the following integers must be divisible by 3?

- a)  $n^2(n + 1)$     b)  $n(n + 1)^2$     c)  $n(n^2 - 1)$     d)  $n^2 + 6n + 9$     e) none of these

7. What is the largest amount of postage in cents that cannot be paid exactly with an unlimited supply of 6-cent and 7-cent stamps?

- a) 15                      b) 29                      c) 32                      d) 41                      e) 43

8. The square  $ABCD$  lies immediately underneath the equilateral triangle  $AEB$ . What is angle  $DEC$ ?

- a)  $15^\circ$                       b)  $20^\circ$                       c)  $30^\circ$                       d)  $45^\circ$                       e) none of these

9. What is the value of  $x + y$  if  $x$  and  $y$  are nonzero numbers for which

$$xy = \frac{x}{y} = x - y?$$

- a)  $-3/2$                       b)  $-1/2$                       c) 0                      d)  $1/2$                       e) none of these

10. What are the last two digits of the number  $2014^{2014}$ ?

a) 16                      b) 36                      c) 56                      d) 76                      e) none of these

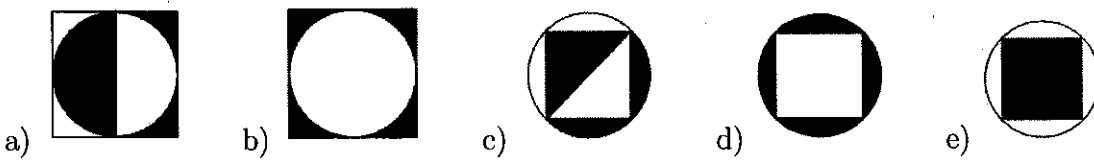
11. Two fair six-sided dice are rolled repeatedly until a sum of either 7 or 10 appears. What is the probability that the sum of 10 appears first?

a)  $7/17$                       b)  $1/2$                       c)  $4/11$                       d)  $5/12$                       e) none of these

12. The following story is often told of the Ancient Greek Mathematician Diophantus:  
 His boyhood was one-sixth of his life. For one twelfth more of his life, he was youth. After another seventh more he married. Five years later, he had a son who ended up living half as long as Diophantus did. Diophantus died four years after his son.  
 How long did Diophantus live?

a) 64 years                      b) 72 years                      c) 80 years                      d) 84 years                      e) none of these

13. The circles in the figures below all have the same radius. They also have either a square inscribed within or circumscribed about them. Which of the shaded regions has the least area?



14. The value of  $8^{\frac{1}{\log_4 8}}$  is which of the following?

- a) 1                      b) 2                      c) 4                      d) 8                      e) none of these

15. Find the solutions of the equation  $x^3 + 4 = 6x$ .

- a)  $2, -1 \pm \sqrt{3}$     b)  $2, -1 \pm \sqrt{5}$     c)  $2, 1 \pm \sqrt{3}$     d)  $2, 1 \pm \sqrt{5}$     e) none of these

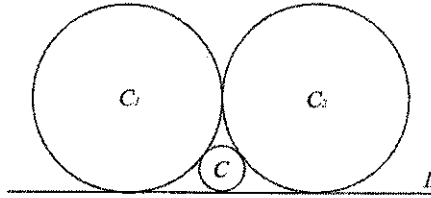
16. What is the 2014th term of the sequence 2, 0, 1, 4, 2, 0, 1, 3, 2, 0, 1, 2, ...?

- a) 1                      b) 5                      c) 6                      d) 7                      e) none of these

17. Find the sum  $\frac{1}{\log_2(3)} - \frac{1}{\log_4(9)} + \frac{1}{\log_8(27)} - \frac{1}{\log_{16}(81)}$ .

- a) 0                      b) 1                      c) 2                      d) 3                      e) none of these

18. Two circles  $C_1$  and  $C_2$  of equal radii are tangent to a line  $L$  and to each other. A third, smaller circle  $C$  is tangent to both  $C_1, C_2$  and  $L$  (see the figure below). If the radius of  $C$  is 3, then the radius of  $C_1$  is



- a) 6                      b) 8                      c) 10                      d) 12                      e) none of these

19. Find the remainder when the polynomial  $x^{100} - 4x^{98} + 2$  is divided by  $x^2 - x - 2$ .

- a)  $x - 1$                       b)  $x$                       c)  $x + 1$                       d)  $2x + 1$                       e) none of these

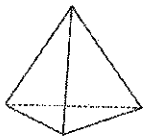
20. Suppose that  $x$  satisfies the equation  $\sin(x) = \cot(x)$ . Compute  $\cos x$ .

- a) 0                      b)  $\frac{\sqrt{3}}{2}$                       c)  $\frac{\sqrt{5} - 1}{2}$                       d)  $\frac{\sqrt{5}}{4}$                       e) none of these

21. The sum of the solutions of the equation  $|x^2 - 4| = |x - 1| - 1$ .

- a)  $1 - \sqrt{15}$                       b)  $1 - \sqrt{17}$                       c)  $2 - \sqrt{15}$                       d)  $2 - \sqrt{17}$                       e) none of these

22. A tetrahedron is a pyramid-shaped regular polyhedron which has 4 equilateral triangular faces. If each edge of a tetrahedron is 1 unit long, how many units tall is the tetrahedron when resting on a flat surface?



- a)  $\frac{1}{\sqrt{3}}$       b)  $\frac{1}{\sqrt{6}}$       c)  $\frac{\sqrt{2}}{\sqrt{3}}$       d)  $\frac{\sqrt{3}}{\sqrt{2}}$       e) none of these
23. How many ways are there to select four of the numbers from the set  $S = \{1, 2, \dots, 10\}$  such that no two of them are consecutive?
- a) 24      b) 35      c) 56      d) 70      e) none of these
24. Let  $p(x)$  be the cubic polynomial  $5x^3 - 6x^2 + K$ . If the three roots of  $p(x)$  form an arithmetic progression, then the value of  $K$  is
- a)  $3/4$       b)  $4/5$       c)  $16/25$       d)  $25/36$       e) none of these
25. Evaluate  $\cos(20^\circ) \cdot \cos(40^\circ) \cdot \cos(80^\circ)$ .
- a)  $1/2$       b)  $1/4$       c)  $1/8$       d)  $1/16$       e) none of these

26. What is the largest prime factor of 2014?

- a) 3                      b) 13                      c) 17                      d) 19                      E) none of these

27. It takes Agatha 90 minutes to mow her mother's yard and her brother Barnabus can mow it in 60 minutes. How long would it take them to mow it together using two lawn mowers.

- A) 36 minutes    b) 150 minutes    c) 40 minutes    d) 42 minutes    e) None of these

28. There are 3 red balls, 2 green balls, and 3 purple balls in a bag. If you take out 3 balls at once, what is the probability that you took out at least one red ball?

- a) 1                      b)  $\frac{27}{28}$                       c)  $\frac{25}{28}$                       D)  $\frac{23}{28}$                       e) none of these

29. Define a sequence  $a_1 = 1$ ,  $a_{n+1} = 6a_n + 1$ . What is  $a_{2014}$ ?

- a) 1                      b) 3                      c) 5                      d) 7                      e) none of these

30. What is the value of  $\log_2(\log_2(\log_2 16))$ ?

- a) 0                      B) 1                      c) 2                      d) 3                      e) none of these

31. If  $\sin \theta \cos \theta = \frac{1}{4}$  and  $-90^\circ \leq \theta \leq 90^\circ$ , then  $\theta =$

- a)  $10^\circ$                       B)  $15^\circ$                       c)  $25^\circ$                       d)  $30^\circ$                       e) none of these