Moments and Centers of Mass

Moments and Center of Mass: One-Dimensional System

Let the point masses \( m_1, m_2, \ldots, m_n \) be located at \( x_1, x_2, \ldots, x_n \).

1. The moment about the origin is \( M_0 = m_1 x_1 + m_2 x_2 + \cdots + m_n x_n \).

2. The center of mass is \( \bar{x} = \frac{M_0}{m} \), where \( m = m_1 + m_2 + \cdots + m_n \) is the total mass of the system.

Moments and Center of Mass: Two-Dimensional System

Let the point masses \( m_1, m_2, \ldots, m_n \) be located at \( (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n) \).

1. The moment about the \( y \)-axis is \( M_y = m_1 x_1 + m_2 x_2 + \cdots + m_n x_n \).

2. The moment about the \( x \)-axis is \( M_x = m_1 y_1 + m_2 y_2 + \cdots + m_n y_n \).

3. The center of mass \( (\bar{x}, \bar{y}) \) is

\[
\bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m}
\]

where \( m = m_1 + m_2 + \cdots + m_n \) is the total mass of the system.

Moments and Center of Mass of a Planar Lamina

Let \( f \) and \( g \) be continuous functions such that \( f(x) \geq g(x) \) on \( [a, b] \) and consider the planar lamina of uniform density \( \rho \) bounded by the graphs of \( y = f(x), y = g(x) \), and \( a \cdot x \cdot b \).

1. The moments about the \( x \)– and \( y \)–axes are

\[
M_x = \frac{\rho}{2} \int_a^b [f(x) + g(x)][f(x) - g(x)] \, dx
\]

\[
M_y = \rho \int_a^b x[f(x) - g(x)] \, dx
\]

2. The center of mass \( (\bar{x}, \bar{y}) \) is given by

\[
\bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m}
\]

where \( m = \rho \int_a^b [f(x) - g(x)] \, dx \) is the total mass of the lamina.