Parametric Formulas

Parametric Form of the Derivative

If a smooth curve $C$ is given by the equations $x = f(t)$ and $y = g(t)$, then the slope of $C$ at $(x, y)$ is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad \frac{dx}{dt} \neq 0$$

The second derivative is given by

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left[ \frac{dy}{dx} \right] = \frac{\frac{d}{dt} \left[ \frac{dy}{dx} \right]}{\frac{dx}{dt}}, \quad \frac{dx}{dt} \neq 0$$

Arc Length in Parametric Form

If a smooth curve $C$ is given by $x = f(t)$ and $y = g(t)$ such that $C$ does not intersect itself on the interval $a \leq t \leq b$ (except possibly at the endpoints), then the arc length of $C$ over the interval is given by

$$s = \int_a^b \sqrt{ \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 } \, dt = \int_a^b \sqrt{\left[ f'(t) \right]^2 + \left[ g'(t) \right]^2 } \, dt$$

Area of a Surface of Revolution

If a smooth curve $C$ given by $x = f(t)$ and $y = g(t)$ does not cross itself on an interval $a \leq t \leq b$, then the area $S$ of the surface of revolution formed by revolving $C$ about the coordinate axes is given by the following.

1. $S = 2\pi \int_a^b g(t) \sqrt{ \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 } \, dt$ \hspace{1cm} Revolution about the $x$-axis: $g(t) \geq 0$

2. $S = 2\pi \int_a^b f(t) \sqrt{ \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 } \, dt$ \hspace{1cm} Revolution about the $y$-axis: $f(t) \geq 0$