

Enumeration and Optimization of Bobbin Lace

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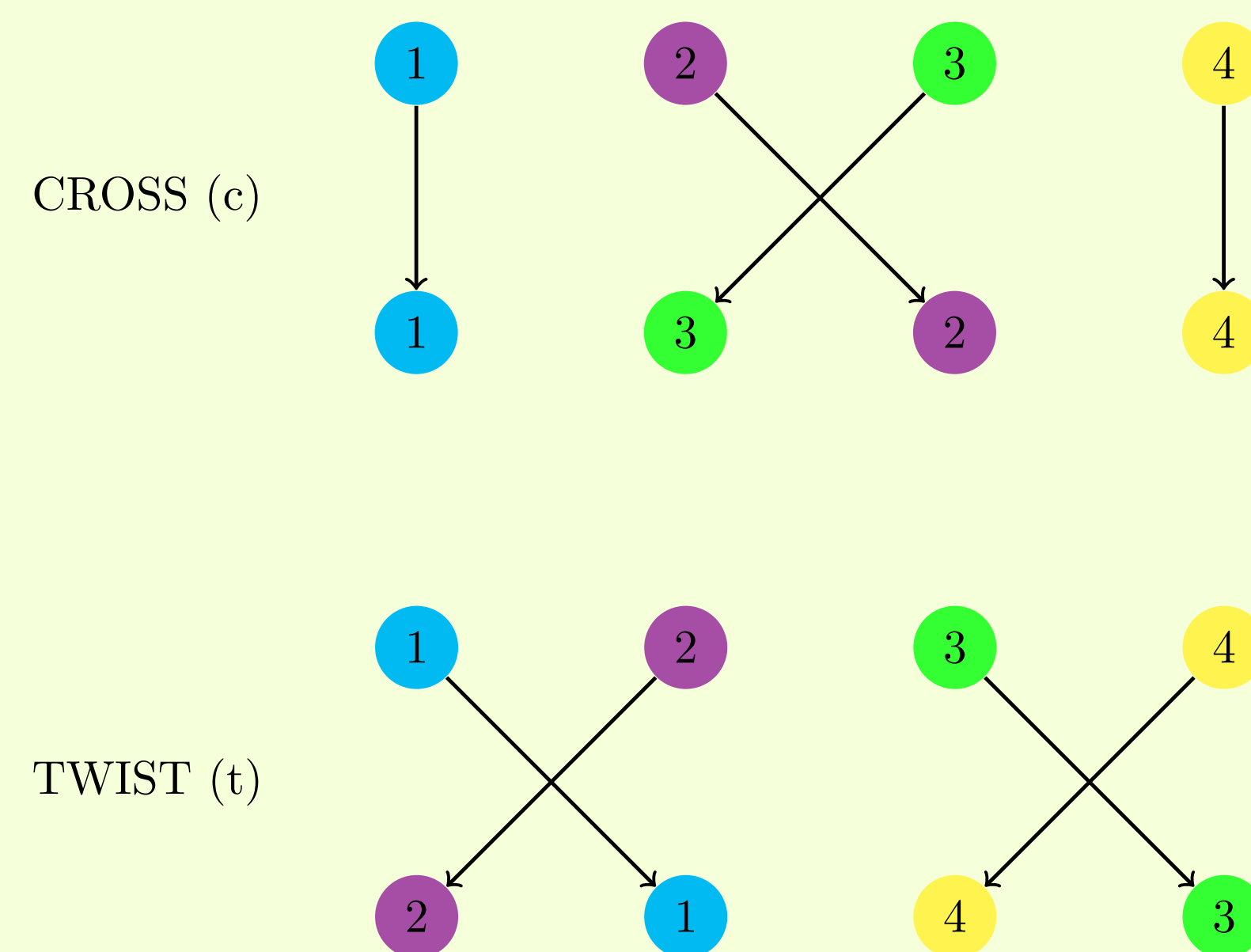
1. Introduction

Bobbin lace is a process of free-form weaving to create a textile. Due to the lack of a wane and weft of a traditional woven textile, this weave can generate less predictable outcomes. This textile art has yet to be explored from a mathematical perspective, and this project aims to not only create the language to discuss bobbin lace mathematically, but to also answer foundational questions to build a base for analysis.

2. Language of Movement

I. Thread Movement

Bobbin lace is created by movements between individual threads akin to a braid. 4 threads are separated into pairs, denoted by the left two threads and the right two. Movement is described in terms of permutations, meaning that two elements move when they trade positions. There are two movements between 4 threads that generate a weave, as pictured below,



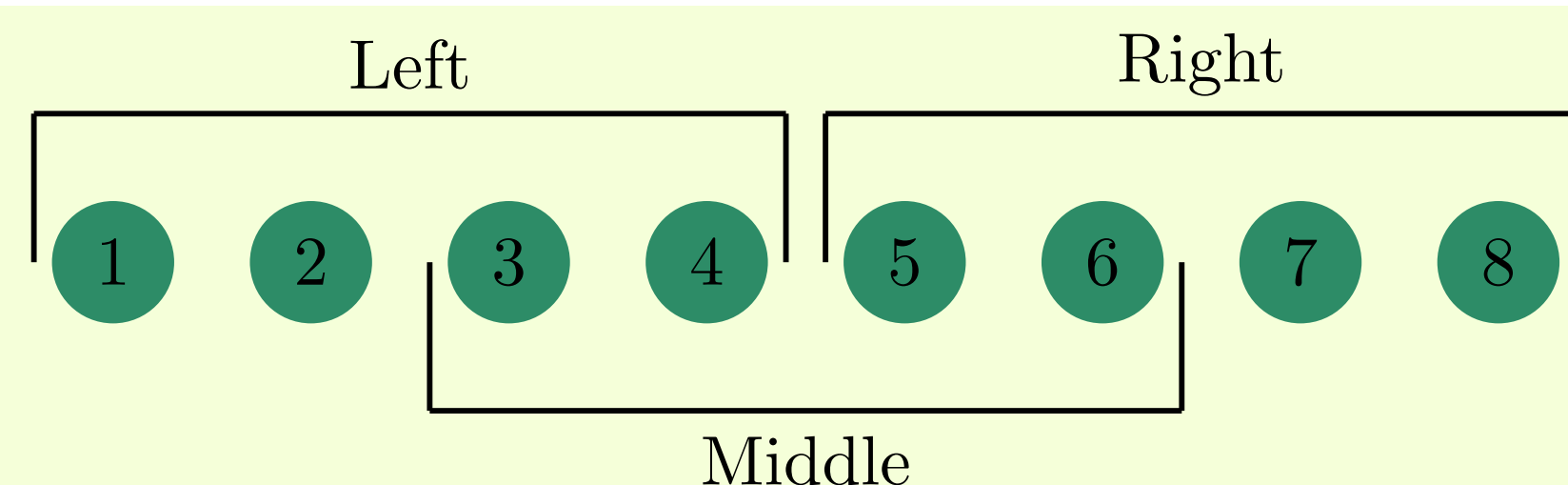
A cross is when the thread in the 2nd position is placed over the thread in the 3rd position, denoted by the permutation (2,3).

A twist is when the threads in the 2nd position and 4th position go over the threads in the 1st and 3rd position, respectively. This is denoted by the permutation (1,2)(3,4).

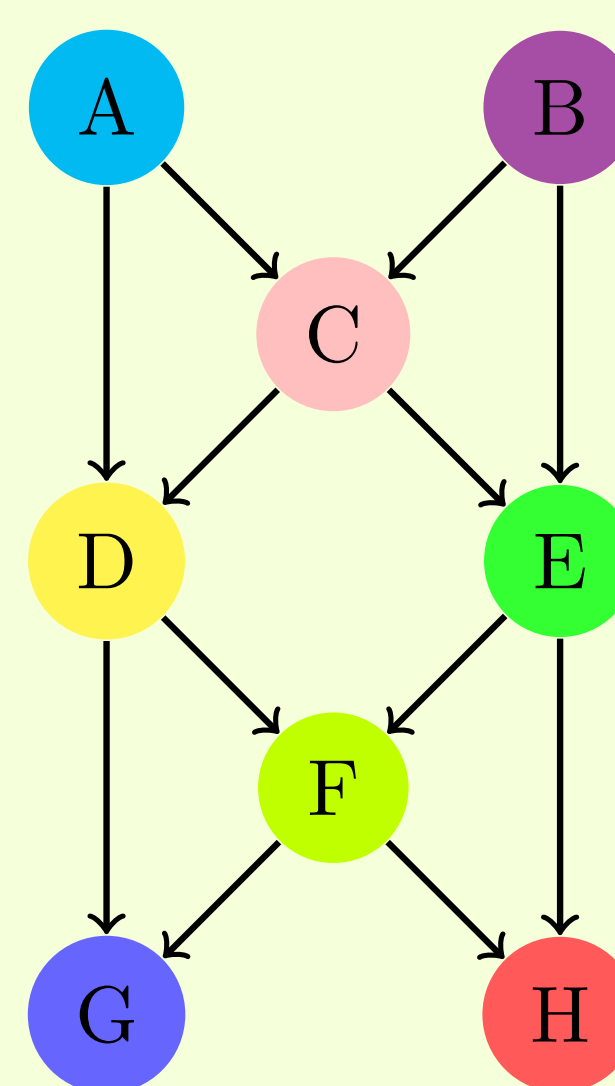
A stitch is defined as a combination of 3 or 4 sequential movements, for example cross, twist, twist would be a single stitch.

II. Stitch Movement

To simplify discussion while remaining meaningful, 8 threads are appropriate. Stitches are made with 2 adjacent pairs. To begin a piece, either the stitch is made in the middle 4 threads or two stitches are made adjacent, using all 8 threads.



Once a stitch is made, the left pair goes to one stitch and the right pair goes to another. For example, for the figure below an arrow denotes the movement of a pair, two threads.



Stitch A has its left pair going to the next left stitch (D), and its right pair going to the middle stitch (C).

Stitch B has its right pair going to the next right stitch (E) and its left pair going to the middle stitch (C).

Stitch C has its left pair going to the left stitch and its right pair going to the right stitch.

3. Permutation Possibilities

Given these standards for movement and stitch interaction, if we limit the parameters to the smallest meaningful form of 8 threads, how many permutations of 8 elements are possible?

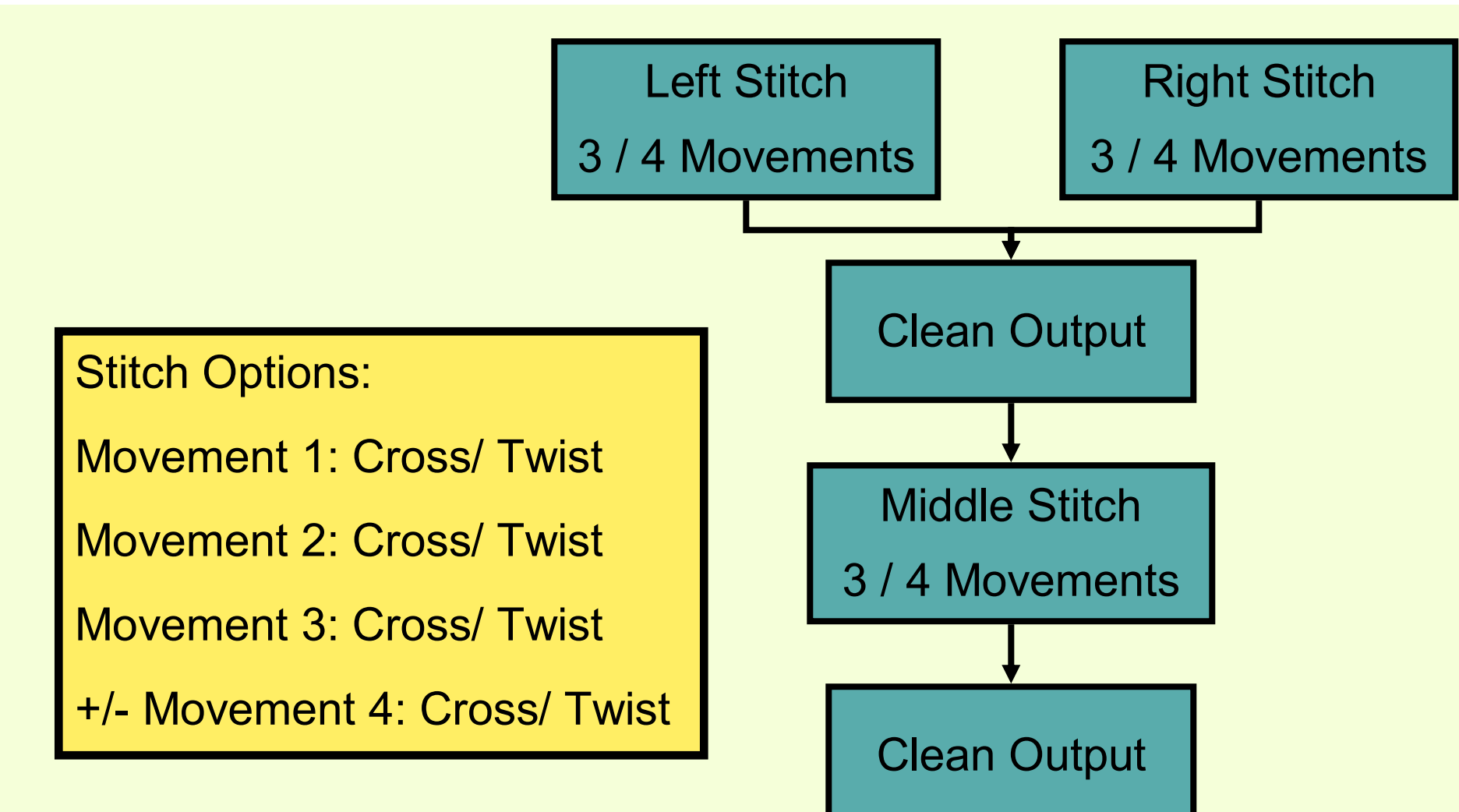
S_8 is the symmetric group that is formed by all possible arrangements of numbers 1 through 8. The number of arrangements is represented by 8 factorial:

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8! = 40,320$$

Is it possible to reach all possible elements of S_8 using the movement structure we have? Furthermore, what is the fewest number of rounds it can be achieved in?

Rounds are represented horizontally, in the figure above, stitches A and B are the first round, C is the second round, and D and E are the third round.

The following figure depicts the general structure for the GAP code. When starting either Left/ Right or Middle, the function takes those appropriate elements and runs through all combinations of 3 or 4 movements. Then, the outputs are run through a "cleaning" process where all repeated outputs are removed.



Stitch Options:
Movement 1: Cross/ Twist
Movement 2: Cross/ Twist
Movement 3: Cross/ Twist
+/- Movement 4: Cross/ Twist

4. Results

Left/ Right starts and Middle starts generate a different number of permutations during certain rounds, as shown below. Removing repeated permutations every round result in the following:

Round	Left/Right	Middle
1	64	8
2	512	512
3	7,488	3,120
4	23,552	23,552
5	40,064	38,976
6	40,320	40,320

5. Conclusion

When trying to generate all possible combinations of 8 elements, starting with a Left/Right or Middle stitch both require 6 rounds of stitches. In 5 rounds or less, 99.82 percent of elements of S_8 are generated, while the remaining 72 combinations are achieved in the 6th round.

This procedure can be applied to larger, more complicated pieces of bobbin lace. We are able to document pieces by tracking the path at each round, as well as optimize a piece for applications in pattern making.

6. References

- [1] Edkins, Jo. "Bobbin Lace - Working Lace." Jo Edkins, Theedkins, <https://www.theedkins.co.uk/jo/lace/eqwork.htm>.
- [2] Gallian, Joseph A. Contemporary Abstract Algebra. 9th ed., Cengage Learning, 2017.
- [3] GAP Release 4.4.12 17 December 2008 Reference Manual. ser. 4.4.12, University of Hawaii, 2008.
- [4] The GAP Group, GAP – Groups, Algorithms, and Programming, Version 4.12.1; 2022. (<https://www.gap-system.org>)